Statistical Analysis for Sovereign Rating Data

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Abstract of the Dissertation

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Being an assessment of a government's ability to liquidate its obligations and a measure of the economic, financial and political situations of an economy, sovereign credit rating becomes progressively important for governments and international financial market. As commonly used rating data, the rating assignments from international rating agencies are viewed as reference of sovereign rating level. Combining country's economic, financial and political data with corresponding rating data, we collect a full set of panel data for a universe of 67 countries, from least developed to developed, covering the period 1989-2016. Following typical statistical method, we use linear methods to analyze the relationship between economic, financial data and sovereign rating data. Then we push the analysis forward concentrating on individual-specific effect. We propose fixed effect approach based on least squares dummy variable (LSDV) model to provide a framework for the analysis and prediction of the sovereign panel data, and for random effect approach, we use linear mixed model for analysis. We apply the statistical method to simulation studies and empirical analysis of sovereign rating and economic panel data. Related conclusion includes the fixed and random effect of country-specific indicator, feedback effect of rating history.

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Chapter 1 Introduction

1.1 Motivation for proposed analysis

Credit ratings are assessments of the relative likelihood that a borrower will default on its debt repayment on principal and interest. When national government acts a part as an issue borrower in the capital market, the credit rating is so-called sovereign rating. Thus, sovereign ratings are assessment of a national government's ability and willingness to repay its debt on time. Since the issuers are national government, which are relevant for the domestic and international financial market, the sovereign ratings play a significant role. To be more specific, the sovereign rating are important in following several aspects. First, sovereign rating are a key determinant of the interest rate and consequently affect the cost of a country borrowing money from the international financial market. Second, sovereign rating affect the ratings assigned to borrowers of the same nationality, like domestic bank and companies. Third, considering the lower bound for the investment risk, the international investor will build their bond portfolio taking the sovereign credit rating into account. Last but not least, as an assessment of the economic, financial and political situation of an economy, the sovereign rating give a measure of the country's development.

Considering the first publication introducing sovereign rating by Moody in 1918, the field concentrated on this topic only have one hundred years. The existing literature devoted to modeling sovereign rating is rather sparse. It begins with Cosset and Roy (1991) and Moon and Stotsky (1993), which introduce the study on sovereign rating determinants, and Cantor and Packer (1996) which present the first systematic analysis of the determinants and impact of the sovereign rating. The main reason leading to lack of literature is that, before 2000, the worldwide rating histories, and even the economics and financial data are not available. Few sovereigns, especially emerging market economics or least developed countries, have ratings history longer than one

or two decades. The estimation used in the existing literature, only taking the developed countries into account, may be not so accurate. Such a statistical analysis has recently become possible as a result of the rapid growth in the size of sovereign rating data, which includes not only the developed countries, like U.S and European countries, but also the least developed countries which locate on Africa, South America and Asia.

In recent year, from 1996, several financial crisis rolled up the whole world. It is worth noting that, some of the crisis are caused by sovereign rating crash. For example, the Greek government-debt crisis happened in late 2009. Considering low GDP growth rate and high government budget deficit, Standard and Poor's, Moody's and Fitch downgraded Greek sovereign rating, and it become the trigger point for Greek debt crisis. Therefore, it is important for both government and financial market to understand what kinds of factor affect more emphasis on sovereign rating.

A typical statistical model in a data analysis study can be denoted as follows where i represents the ith individual and t stands for the time,

$$y_{i,t} = x'_{i,t}\beta + \alpha + \epsilon_{i,t}$$

To study the determinant factor behavior, we can estimate the "global effect" β which model homogeneity. To study the country-specific behavior, we include the "individual effect" α which model heterogeneity and thus denote it as α_i. To deal with the individual effect α_i, we use two major approaches in the literature and in each approach we estimate β separately. The first approach is fixed effect model, while the second is random effect model.

Fixed Effect Model

To deal with the country-specific effect, we introduce fixed effect model as first direction.We consider the fixed effect model

$$y_{i,t} = x'_{i,t}\beta + \alpha_i + \epsilon_{i,t}$$

where α_i is a fixed effect, which is treated as an unknown parameter to be estimated. Therefore, no specific distributional assumption for α_i is required.

The fixed effect model is commonly used by macro-economists. Generally speaking, the fixed effect model is more appropriate than a random effect model in following two reason. First, if the individual effect represents omitted variables, these country-specific characteristics are possibly correlated with the other regressors. Second, the typical macroeconomic and financial data set is likely to contain most countries of interest, and consequently, might be not a random sample. For example, when we take a political variable, OECD membership, into consideration, an OECD country may be interacted by other countries in OECD. Thus, we can not neglect the fixed effect approach in our empirical analysis.

Random Effect Model
Random effect model assumes $\alpha_i \overset{i.i.d}{\sim} N(0, \sigma_\alpha^2)$ in the statistical model to be unknown country-specific random effects. It's commonly called error-component model since the error comes from two parts, the random effect α_i and the random error $\epsilon_{i,t}$.

1.2 Outline of Dissertation

We divide the dissertation into the following chapters. In Chapter 2 we investigate the history of sovereign credit rating, determinants of sovereign rating commonly used in existing literature and the current statistical approaches.

In Chapter 3 we show and describe the explanatory and response variable in our proposed method. In particular, we use numerical rating level as response variable, take economic and financial data commonly used in current study into consideration and introduce several dummy indicator to represent the geographic or political situation.

In Chapter 4 we propose the statistical model approach via LSDV and LMM for the analysis of panel data where cross-sectional and individual-specific information can be estimated better.

In Chapter 5 we apply the fixed effect and random effect approach proposed in Chapter 4 to model numerical sovereign rating level that are jointly exposed to the explanatory variables: economic, financial and political variables, which

we introduce in Chapter 3, in a simulation study. Moreover, we introduce rating history movement as feedback effect analysis. Finally, Chapter 6 gives the concluding remarks.

Chapter 2 Background and Literature Review

2.1 History of Sovereign Rating

The terminology, sovereign ratings, is first introduced by Moody's Investors Service in their publications *Moody's Analyses of Investments* and *Moody's Manual* in 1918 (Moody, 1918). Different from the definition, from both qualitative and quantitative aspects, being used nowadays, sovereign ratings were setup for measurement about relative creditworthiness of bond issuers, central government, at that time. In 1920, Moody's provided its categorical measurement of creditworthiness, which is much simpler than Moody's criteria used currently, and established a rating range, from the high figure *Aaa* downward, which contains *Aaa*, *Aa*, *A*,*Baa, Ba* and *Lower* (Moody, 1920). In the following two or three years, Poor's, Fitch and Standard Statistics gave their rating system similar to Moody's. At that period, the country coverage was very limited: the United States and Canada led the rating level with *Aaa* and *Aa*, great nations like Great Britain and France enjoyed the high credit level because of their great age and vast resource, while the dilapidated countries of Balkan States and Turkey with lower rating suffered from years of political turmoil. The coverage of rating levels and countries extended in the prominent publication of Moody's in 1922, which provided the marked expansion and accurate definition of rating level system (Moody, 1922). At that period, the credit rating agencies concentrated on some certain bonds issued by one sovereign, not the country itself, and provided their rating to the bonds, for example, shown in Table 2.1:

Notice that, under the rating criterion, the accurate meaning of sovereign rating, at 1920s even long period later, is sovereign bonds rating, which is established for bonds, not sovereign issuers. So, the rating categories only described the feature of bonds.

As shown in the introduction section of Moody (1922), entitled as "Key to the ratings". The rating scale included nine categories: *Aaa, Aa, A, Baa,*

Table 2.1: Details of some U.S debts as of June 30, 1920

Title of Bond	Rate of Interest	Date of Issue	When Redeemable or Payable	Amount	Rating
Consols of 1920	2's	1900	Payable after April 1, 1930	$646,250,150	Aaa
Loan of 1925	4's	1895-1895	Payable after Feb. 1, 1925	$162,315,400	Aaa
Conversion Bonds	3's	1916-1917	Payable 30 years from issue date	$28,894,500	Aaa
First Liberty Loan	3.5's	1917	Redeemable after June 15, 1932 Payable after June 15 1947	$1,410,074,400	Aaa
First Liberty Loan Conv.	4's	1917	Redeemable after June 15, 1932 Payable after June 15 1947	$65,803,050	Aaa
First Liberty Loan Conv.	4.25's	1918	Redeemable after June 15, 1932 Payable after June 15 1947	$473,089,200	Aaa

Source: Moody (1920)

Ba, B, Caa, Ca and *C*, while the other rating agencies followed similar setup. With these rating scales, rating agencies furnish a tool to classify the security and stability of particular government bonds and to measure the ability and willingness to repay the debt, both principal and interest. Notice the rating system, with consideration of governments' situation, provided measurements about ability and willingness, not reflectors of bonds prices or bond maturities. For example, the ratings didn't imply that the bond price may decline after rating decreasing as a result of changes in demand and supply, economic crises, etc.

2.1.1 Development of Rating Scale and its meaning

Back to the rating categories, first introduced by Moody's in sovereign rating, we summarize from Moody (1922):

Aaa : The ratings are classed in the group of first grade issues, so far as "intrinsic strength and security are concerned", "assurance of the prompt payment of principal and interest" and "permanent intrinsic worth of each issues".

Aa : The bond in this level indicate that they are "strong investment and generally fundamentally secure", subject to "some qualification in security or stability", and come with "good past record".

A : A rating comes in the general broad group known as "good", which are down the investment scale. While, the security is "permanent and

well-demonstrated".

Baa : These kinds of bonds "carry some speculative quality", may "not have sufficient permanent stability to warrant a higher standing for its obligations" and they are "liable to become largely speculative".

Ba : Many of issues under this rating "possess some investment quality, yet all carry a distinctly uncertain tinge", and "can't be regarded as at all attractive from the standpoint of real security". A security of this type is "purchased for its speculative possibilities rather than its investment quality"

B : Bonds carrying this rating are "in imminent danger of defaulting".

Caa : "The obligations of dangerously weakened communities".

Ca : The bond is an obligation that "very little value if left of the security" and "there is little or no hope of any substantial improvement short of partial repudiation"

C : A rating "are apt to become practically worthless". A bond of this rating is typically in default, i.e, has defaulted or appears certain to default within the very near future.

In the following several years, the other three credit rating agencies (CRAs), Fitch, Standard Statistics and Poor, provided their classification on sovereign bonds by their own setup. Fitch and Standard Statistics define the rating scales with reference based on their corporate bonds assessment. Fitch set up twelve categories: *AAA, AA, A, BBB, BB, B, CCC, CC, C, DDD, DD,* and *D*, while Standard Statistics introduced fourteen ratings: *A1+, A1, A, B1+, B1, B, C1+, C1, C, D1+, D1, D, E,* and *F* [Fitch, 1924; Standard Statistics, 1924]. Another rating agency, Poor, provided rating scales, which are different from their corporate scales, as follows: *A*****, A****, A***, A**, A*, A, B**, B*, B, C**, C*,* and *C*. Although the rating scale symbols provided by these four CRAs were different from each other, they all classify the rating level into several groups. For instance, Fitch classified all three *A*-level, *AAA, AA, A*, to high grade bonds, and subsequently they assigned three *B*-level to lowest-grade investment bonds, three *C*-level to speculative bonds and three *D*-level to bonds in default. Similarly to Fitch's setting, Standard Statistics and Poor classified the rating scales to investment grade, speculative grade and default. Notice that, the rating from these four CRAs were not strictly

comparable as they set up at beginning, easily concluded from the facts that the number of rating levels from agencies were different from each other.

In 1941, Standard Statistics and Poor merged into a new rating agency, Standard and Poor (S&P), then it followed the setting of Fitch, while replaced *DDD, DD, D* with *D, SD*, *SD* here for selective defaults. One remark happened in 1975, that the U.S. Securities and Exchange Commission designated nationally recognized statistical rating organizations (NRSRO), and the members of NRSRO were *Moody's Investors Service*, *Standard & Poor's* and *Fitch Rating*. In 1970s-1980s, S&P and Fitch began to refine their rating categories by introducing three sub-rating, a plus (minus) sign, to *AA-CCC* rating in order to represent the higher (lower) creditworthiness. In 1986, Moody's introduce a numerical sub-rating to its rating. As a result of these refinements, they got the rating system, which is still using nowadays. We summarize the system in the following, Table 2.2

Table 2.2: Current system of Moody's, S&P and Fitch ratings

Category	**Interpretation**	**Moody's**	**S&P**	**Fitch**
Investment Grade	Highest quality	Aaa	AAA	AAA
	High quality	Aa1	AA+	AA+
		Aa2	AA	AA
		Aa3	AA-	AA-
	Strong payment capacity	A1	A+	A+
		A2	A	A
		A3	A-	A-
	Adequate payment capacity	Baa1	BBB+	BBB+
		Baa2	BBB	BBB
		Baa3	BBB-	BBB-
Speculative Grade	Likely to fulfill obligations, ongoing uncertainty	Ba1	BB+	BB+
		Ba2	BB	BB
		Ba3	BB-	BB-
	High-risk obligations	B1	B+	B+
		B2	B	B
		B3	B-	B-
	Extremely high risk bond or investment	Caa1	CCC+	CCC+
		Caa2	CCC/CCC-	CCC/CCC-
		Caa3	CC/C	CC/C
Default	Defaulted	C	SD/D	RD/D

Accompanying with the rating scales being set up, one more remark should be borne in mind that the ratings attributed to all government bonds, both

domestic and foreign, come from the standpoint of the American investor. The "big-three", Moody's, S&P and Fitch, are U.S.-based agencies, while Fitch headquarter in New York though belong to 100 percent to the French holding company Fimalac until 2006. They provide the rating based on the point of investors who think and act in local currency, "dollars". The other agencies based in different country, such as JCR, DBRS or Dagong Global, which will be discussed in the following part, start from local investors' point, in their local currency.The national government, the largest issuers and buyers in the global capital market, generally seek rating on foreign currency obligations, which are different from CRAs' standpoint. One important reason is that foreign currency bonds were more preferred by international investors than domestic currency bonds.

In the following section, we will introduce the local and foreign currency rating policy. What's more, considering the maturities of different bonds, we present the rating policies on short-term and long-term bonds.

2.1.2 Sovereign Rating Policies and Rating Process

After the first sovereign bond rating were introduced by Moody in 1918, the credit rating agencies persevere in seeking the complete and comprehensive rating system to follow the rapid development in international credit market. Initially, the rating system was set up for sovereign bonds exclusively, and the government, sovereign issuers, were not rated. In the 1980s, credit rating agencies began to rate sovereign bond issuers. Generally speaking, the sovereign ratings are opinions about the issuer's overall economic and financial capacity to pay its obligations, both principal and interest, and should not be applied for any specific bonds.

Keeping pace with the development in financial market, the CRAs proceed to complete their rating policies. Comparing with the rating, which only set up for any certain bonds in local currency in 1920s, the current rating policies are more comprehensive and reasonable. We will introduce the policies following the timeline of rating system development, and they contain: Short-term ratings, rating outlooks and watches, country ceilings. [1]

[1]Without any specific illustration, the remaining of this dissertation uses the long-term foreign credit ratings of sovereign issuers when analyzing the sovereign ratings, especially in

Generally speaking, sovereign ratings contain two aspects: [2]

- Government bond rating (may comes with outlooks): Aims at measuring the risk that a government may default on its own obligations in either local or foreign currency. It takes into account both the ability and willingness of a government to repay its debt in a timely manner.
- Local and foreign currency ceilings: Aims at assessing possible governmental interference on the capacity of other economic agents to repay debt. Foreign currency country ceilings assess transfer risk that foreign currency debt payments and deposits may be restricted by the government. The local currency deposit ceiling reflects the risk of a disruption or shutdown of the domestic payments system as well as the ability of monetary authorities to support banks during possible banking crises. The local currency ceiling indicates, which based on the basis of economic, financial and structural criteria, the highest rating for an issuer domiciled within a given country. These ceilings cap, under certain condition, the ratings of specific securities or issuers.

Table 2.3: Overview of Sovereign Ratings

	Local Currency	**Foreign Currency**
Bond Ratings	Local Currency Government Bond Ratings reflect CRAs' opinion of the capacity and willingness of a government to generate revenues in it own currency to repay its debt to bond holders on a timely basis .	Foreign Currency Government Bond Ratings reflect CRAs' opinion of the capacity and willingness of a government to mobilize foreign exchange to repay foreign currency-denominated bonds on a timely basis.
Ceilings	Local Currency Ceiling summarizes the general country level risk (excluding foreign-currency transfer risk) that should be taken into account in assigning local currency ratings to locally-domiciled obligors or locally-originated structured transactions.	Foreign Currency Ceiling reflects the risk of governmental interference at times of external payments crisis, i.e. foreign currency transfer risk. The ceiling is defined by the probability that a government would resort to a moratorium should it default. It is determined by multiplying the implied default risk associated with existing foreign-currency government bond ratings by the risk that a moratorium would be used as a public policy tool.

Source: Moody (2004)

Traditionally, sovereigns issue debt in local currency or in foreign currency. In 1910s, the period that sovereign rating just were established, the CRAs only assigned ratings to bonds denominated in US dollars or British pounds. Then in 1990s, they covered sovereign issuers and assigned foreign currency rating to them.

the data and methodology sections.

[2]Source: Moody's Corporation (2001)

Short-Term Ratings

Short-term ratings are assigned to obligations with an original maturity of thirteen months or less. As a contrast, long-term ratings are assigned to issuers or obligations with an original maturity of one year or more. Accompanying with the development of financial market, more and more sovereign bonds come into the market. Besides the long-term bonds, such as mentioned in Table 2.1, there came some short-term bonds. Moody's issued first short-term ratings to sovereigns to satisfy the willingness of monitoring short-term bonds in market as early as 1971, Standard and Poor's and Fitch issued their short-term ratings in 1975 and 1995, respectively. Similar with the rating category setting for long-term bonds, the CRAs provide their rating category system for short-term bonds. Moody's short-term rating scale is displayed as follows (Moody, 2004):

- **Prime-1**: Issuers (or supporting institutions) rated Prime-1 have a superior ability to repay short-term debt obligations.
- **Prime-2**: Issuers (or supporting institutions) rated Prime-2 have a strong ability to repay short-term debt obligations.
- **Prime-3**: Issuers (or supporting institutions) rated Prime-3 have an acceptable ability to repay short-term obligations.
- **Not Prime**: Issuers (or supporting institutions) rated Not Prime do not fall within any of the Prime rating categories.

The following Table 2.4 indicates standard linkage between long-term and short-term rating scales (Moody, 2004):

S&P and Fitch introduce different rating symbols and establish a link between short-term ratings and long-term ratings (see Table 2.5).

S&P and Fitch provide similar rating symbols for short-term bonds. The A-level (A-1+, A-1, A-2 and A-3) by S&P and F-level (F1+, F1, F2 and F3) by Fitch indicate a investment grade for short-term credit quality, which is derived from long-term bonds rating. The speculative grade obligors are rated as the B and C short-term rating, since they has a relatively weaker capacity

Table 2.4: Standard Linkage between Long-Term and Short-Term Rating Scales

Short-Term Rating	**Long-Term Rating**
Prime-1	AAA AA+, AA, AA- A+, A, A-
Prime-2	A- BBB+, BBB, BBB-
Prime-3	BBB+, BBB, BBB-
Not Prime	BB+, BB, BB- B+, B, B- CCC+, CCC, CCC- CC, C

Table 2.5: Standard Mapping of Short-Term Ratings to Long-Term Ratings

Standard and Poor's		Fitch	
Long-term Rating	Short-term Rating	Long-term Rating	Short-term Rating
AAA, AA+, AA, AA-	A-1+	AAA, AA+, AA, AA-, A+	F1+
A+, A	A-1	A+, A, A-	F1
A-, BBB+, BBB	A-2	A-, BBB+, BBB, BBB-	F2
BBB-	A-3	BBB, BBB-1	F3
BB+, BB, BB-, B+, B, B-	B	BB+, BB, BB-, B+, B, B-	B
CCC+, CCC, CCC-, CC, C	C	CCC+, CCC, CCC-, CC, C	C
SD, D	SD, D	RD, D	RD, D

to meet commitment, or even are vulnerable to a short-term default risk. The D-level indicates, the CRAs believe that the obligors has partially or generally defaulted on the bonds.

Rating Outlooks and Watches

Rating outlooks and watches are mutually exclusive. Outlooks indicate the expected direction of rating movement. They reflect economic or financial trends which did happen or will happen may cause a rating movement, but which may do so if such trends continue. Standard and Poor's issued first rating outlooks for sovereign entities in 1989, Moody's and Fitch did the same in 1997 and 2000, respectively.

There are four categories of rating outlooks: positive, negative, stable and developing (termed "evolving" by Fitch). A positive rating outlook indicates an upward trend on the rating scale. Conversely, a negative outlook, for example, points to a higher likelihood of rating deterioration. Positive or negative rating outlooks do not imply that a rating change is inevitable. Developing outlooks typically indicate circumstances that could move the rating either up or down. Some ratings have no outlook assigned. Stable outlooks mean that the rating may not change, while ratings with stable outlooks can be raised or lowered without a prior revision to the outlook [3].

It is worth noting that ratings in the C-categories (Caa, Ca, C by Moody's and CCC, CC, C by S&P and Fitch) typically do not carry outlooks since the volatility of these ratings is very high and outlooks would be of limited informational value. Defaulted ratings do not carry Outlooks.

Rating watches indicate view on the expected direction of the rating movement in the short term and becomes applicable when there is an event, the credit implications of which are either unclear or not fully ascertainable immediately. Positive watches indicate that a rating could stay at its present level or potentially be upgraded, negative watches indicate that the rating could stay at its present level or potentially be downgraded, and developing watches are given if ratings may be raised, lowered or affirmed.

Country Ceilings

The "country ceiling" indicates the highest ratings assigned to the sovereign issuer rating of an entity subject to the monetary sovereignty of that country or area. Since the sovereign bonds can be issued in local or foreign currency, there are two kinds of ceilings: foreign currency ceiling and local currency ceiling.

Foreign Currency Ceiling

Foreign currency ceiling reflects, the risk of governmental interference at times of external payments crisis and the degree of interference that sovereign action

[3] "*Rating Definition*" (Fitch, 2018)

can impose on the capacity of a non-sovereign to meet contractual obligation. The rationale for the existence of the foreign-currency ceiling, as well as the foreign-currency bond rating, is that all domestic issuers are potential subject to foreign currency transfer risk, in other words, they may not be able to convert local currency into foreign currency on time to repay the debt.

We can determine the foreign currency ceiling with two parts:

1. **Foreign Currency Bond Rating**, the government's probability of default in foreign currency.
2. **Risk of Moratorium**, the probability that the government will impose a moratorium when it confront with a crisis.

Generally speaking, the ceilings are higher than bond rating in one or two notches.

Local Currency Ceiling

Local currency ceiling summarizes the general country level risk (excluding foreign-currency transfer risk) that should be taken into account in assigning local currency ratings to locally-domiciled obligors or locally-originated structured transactions. Moreover, the ceiling indicates the highest level of rating assigned to the financial strongest obligations in the country.

Typically, local currency ceilings are high, and sometimes much higher than the government's local currency bond rating.

Rating Process

The current sovereign rating process is developed by the CRAs which built up the features and structure in the early 1990s. After years of development, the rating process got a complete framework. We summarize a general process from CRAs and list as follows:

1. **Rating Application**:The rating process typically begins with a credit rating request from the sovereign bond issuers. After a meeting which

introduces rating process, methodology and products, and moreover, the discussion about issuing organization, the rating application will be set up. Once the issuers completed the application, a group of analysts will be assigned to issuers. It is worth noting that there will be a internal conflict compliance check to ensure analysts do not have any conflict interest to issuers.

2. **Information Collection**: Sovereign bond-relevant financial and non-financial information will be collected after the application. The precise list of information may vary according to the sector and market information. Typically, it includes historical and projected financial information, industry and economic data, peer comparisons, and details on planned financing. And the analytical team reviews information from both public and nonpublic sources. For some rating agencies, they hold management meeting with issuers, and the discussion at the meeting focus on credit strengths and weaknesses, trends in industries and sectors , national political and regulatory environment, management and debt structure.

3. **Analysis and Committee**: Once information has been organized, the analyst team will conduct by applying the relevant credit rating methodologies which are differ by agencies. Besides, the analysis is based on quantitative factor in collected information, it's also based on qualitative factors, such as the institutional or governance framework, the financial strategy of the rated entity and, generally, the experience and credibility of management. After completion of the analysis, rating agencies will formulate the a rating committee to consider their recommendation, Rating Committees are a critical mechanism in promoting the quality, consistency and integrity of our rating process. Credit ratings are determined only through rating committees, by a majority vote of the rating committees members, and not by any individual analysts. Then the rating committee votes on the rating recommendation and reaches a rating decision.

4. **Notification and Publication**: After the rating committee reaches a rating decision, the rating agencies typically contacts the sovereign bond-issuer or its designated agent to inform them of the committees decision and the key elements underlying it. After notifying the Issuer, the new ratings will be published and disseminated to the media. Some rating agencies accept a request from an issuer to appeal a rating decision. With that meaningful new information presented, the analyst team will review the additional information and vote.

5. **Surveillance**: As the last step, once a credit rating has been published, the rating agencies will maintain surveillance on an ongoing basis and monitor the credit rating which will be modified as necessary in response to changes in opinion of the creditworthiness. With periodic meetings as part of the ongoing surveillance process, the rating agencies can continuously monitor and update rating policy by apprising of changes in the Rated Entitys plans and discussing new developments.

2.2 Determinants in Sovereign Rating

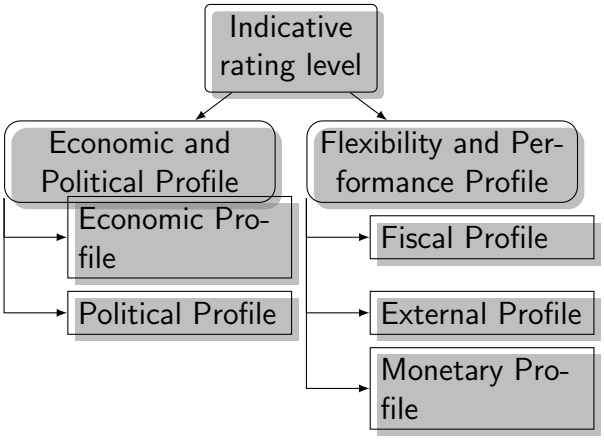
To assign the rating level of a certain sovereign bond, which exclusively applies to the creditworthiness of a government's bond to private creditors, the credit rating agencies concentrate on a broad range of factors. In the rating criteria, which is provided by CRAs, it take into consideration not only the development of economics and degree of interdependence with international market, but also solvency aspects such as the stability of political environment and social cohesion. What's more, unlike corporate bonds which release claim when a default occurs, the sovereign bonds have lower degree interdependence of bond-market.Therefore, it is worthwhile noticing that sovereign bond rating should relate to several potential factors, such as external solvency, default history, and membership in international cooperation organization.

Considering the miscellaneous factors mentioned above, to assess the credit rating of sovereign is a complicate problem. Referring to the rating criteria, numerous economic, financial, social, and political factors are listed not only qualitatively, but also quantifiable. Obviously, for qualitative factors, identifying the relationship between rating and factors is difficult. Even for the quantifiable factors, the CRAs provide little guidance as to the relative weights, which they assign to each factor.

Cantor and Packer (1996), and Haque et al. (1996), not only discuss the rational for, but also measure the relative significance of, several economic and financial factors which are repeatedly cited by CRAs' rating criteria as rating determinants. Ul Haque et al. (1998) discuss the relative importance of political factors in rating. Moreover, Moody's Corporation (2001), Fitch (2012), and Standard&Poor (2017) provide the rating criteria and definition.

We summary that the foundation of sovereign credit rating are built on five assessment, each of which may contains a set of quantitative and qualitative consideration (See Figure 2.1):

Figure 2.1: Five Key Area to Determine A Sovereign Rating



Source: Standard&Poor (2017)

Identifying the assessments mentioned above as a set of quantitative and qualitative factors, we reorganize in five main profile. And in the following parts, we will provide description of each factor in several perspectives, such as economic definition and meaning, the relative significance of sovereign rating, and existing literature or historical events.

- **Economic Profile**: Economic development, prosperity and growth trend
- **Political Profile**: Political stability, international cooperation
- **Fiscal Profile**: Government fiscal performance
- **External Profile**: External liquidity and solvency
- **Monetary Profile**: Monetary sustainability

2.2.1 Economic Profile

Referring to the history of development in sovereign rating, a related-high rating level always couple with a well-developed economic structure. On the

contrary, economic structure in some countries lacks momentum, even suffers recession, and as a consequence, the sovereign bond in corresponding country may defaults in a high probability. To summarize the trait of high rating countries, we can conclude that, wealthy, diversified economic entities, coupled with sustained economic growth, can be assigned related-high rating level. Analysis with causality, a wealthy, sustained-growth economic structure, provides the sovereign government with a strong income and tax base, enhances its fiscal and monetary policy flexibility, and ultimately boosts its debt-bearing capacity.

Summarizing the economic prospects, which leads to distinguishable rating level, we can obtain two aspects:

- *Structure* encapsulate economic prosperity and diversity
- *Growth* capture the dynamic trend of economy

To concatenate the economic concept with quantitative factor, we can aim to seek several proxies to describe. As the first factor, real GDP can be viewed as a comprehensive proxy for the level of development of the economy. Intuitively speaking, a rich debtor have a better reputation and a lower credit risk than a poor one. Various literature show that GDP is the most important explanatory variables for sovereign ratings. Secondly, taking the concept of population into consideration, GDP per capita is an integrated proxy, which is related to, not only economic development, but also population. Besides, the emphasis on the level of prosperity, the growth also acts as a significance role. The growth prospects not only take a measure of capacity to generate government revenue, but also reveal development potential. Therefore, we take the rate of growth of real GDP per capita as the third proxy.

To be more specific, we use the following factors as our determinants in analysis:

- **Real GDP**

  Real GDP is most prominent measure of sovereign income level. With higher GDP, a country has broader potential tax and fiscal bases, and consequently has great ability to repay debt. An important statistic that indicates whether an economy is expanding or contracting, GDP can be tracked over long time and used in measuring a nations economic

growth, as well as in determining if an economy is in recession. Besides that GDP can measure a nation's economic environment, it can also serve as a proxy for the political stability and other important factors.

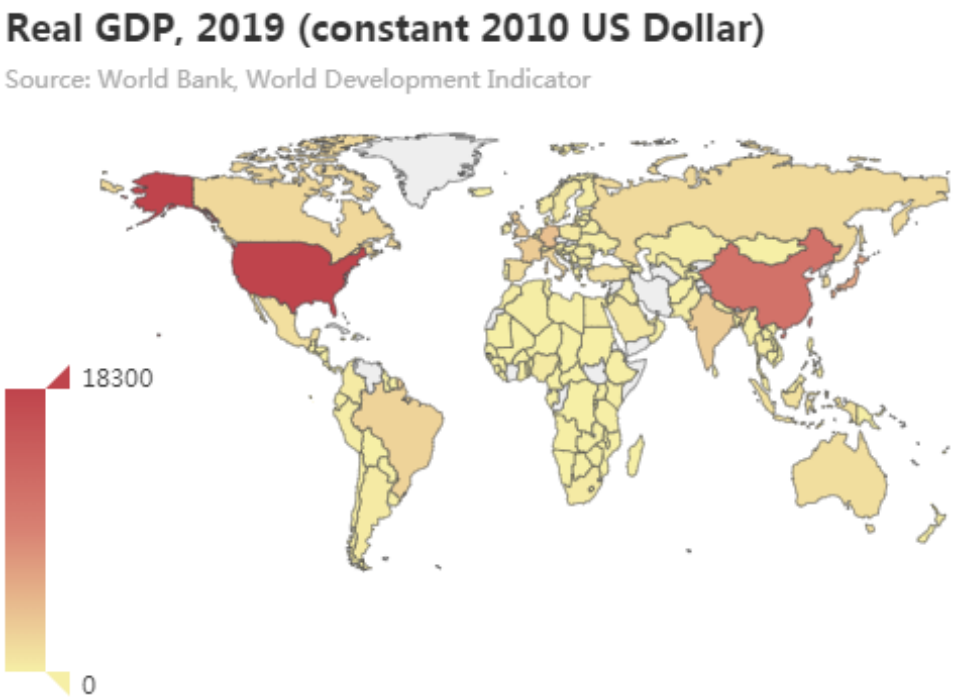


Figure 2.2: Real GDP by Country (US Dollar in billions), 2019

As a sample, Figure 2.2 provide a worldwide version about real GDP in 2019.

- **GDP per capita**

  GDP per capita is another important measure of income level. Different from GDP, which refers to the total value of final goods and services produced within a country or region in a specific time period, GDP per capita is a better indicator of the change or trend in a nations economic performance over time, since it adjusts for population which is different among all the countries.

  As a sample, Figure 2.3 provide a worldwide version about GDP per capita PPP in current international dollar in 2019.

- **GDP per capita Growth**

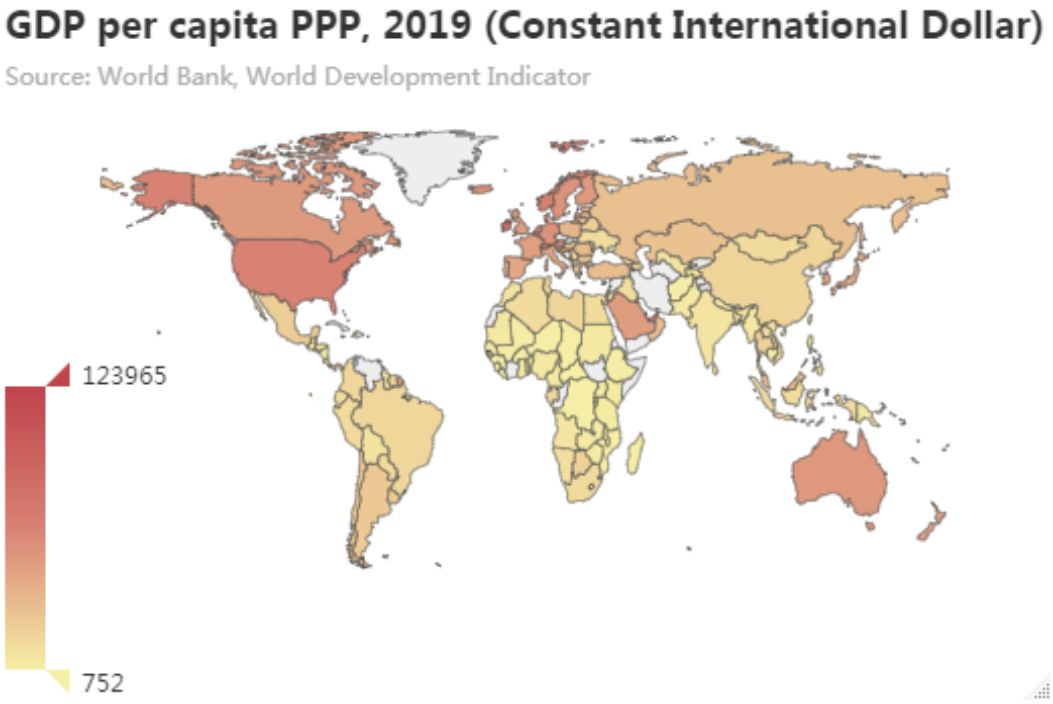


Figure 2.3: GDP per capita by Country, 2019

The key measure of economic growth is real per capita GDP growth. With high economic growth rate, a country's existing debt burden can easier to be service over time. The growth rate of GDP is often used as an indicator of the general health of the economy. In broad terms, an increase in GDP is interpreted as a sign that the economy is doing well. When real GDP is growing strongly, the whole economic performance is getting better, for example, employment is likely to be increasing and so is the government tax. When GDP is shrinking, as it did in many countries during the recent global economic crisis, government revenue often declines.

As a sample, Figure 2.4 provide a worldwide version about GDP per capita growth in 2019.

- **Economic Development** Besides the GDP per capita, which can measure the level of development, the rating agencies also use other factors to reflect the relationship between development and risk. Typically they use two different indicator variables to measure the causal relationship, which make the analysis more comprehensive. One is industrialized clas-

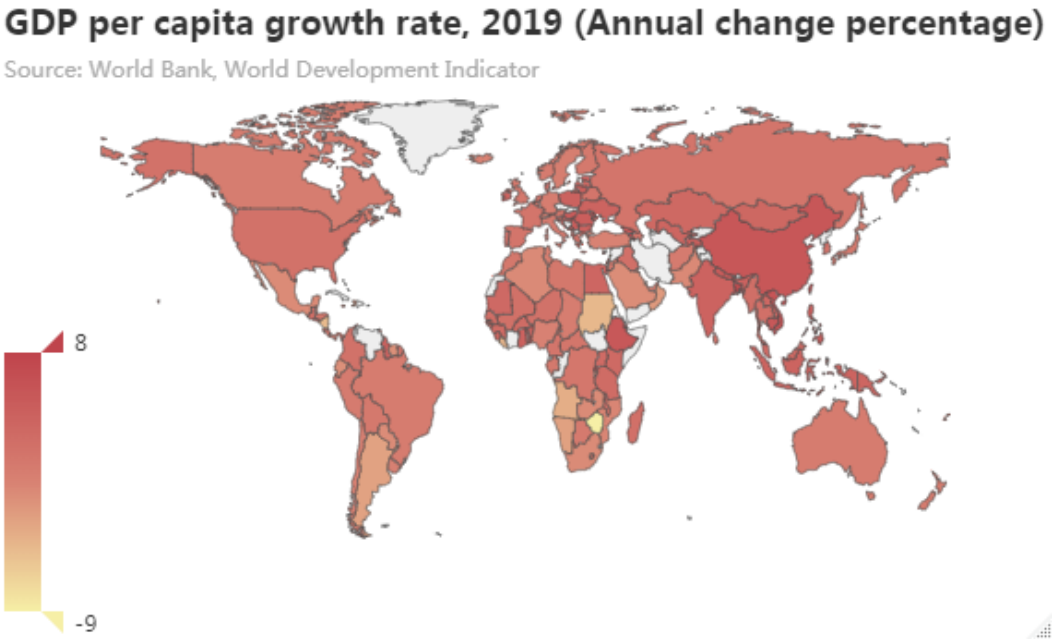


Figure 2.4: GDP per capita growth rate by Country, 2019

sification, another is Least Developed Countries (LDCs) classification.

2.2.2 Political Profile

When rating agencies assign the sovereign credit ratings to different debtors, political factors play a crucial role in rating process. Political factors, or the level of political stability, not only enhance the development of economic prosperity, but also are closely correlated with sovereign debt default. Aiming to capture political event risk, the political stability is taken into consideration, along with some assessment of global integration, geopolitical stability, and policy flexibility. Institutional and geopolitical considerations attempt to measure the promotions and constraints on a sovereign's credit fundamentals by policy-making. Factors considered in the political profile includes: levels of economic cooperation and development, geopolitical engagement, and economic-cultural origin.

- Geopolitics

- **Group of Seven** or **G7**, an indicator variable.
  The Group of Seven or G7 is a group consisting of Canada, France, Germany, Italy, Japan, the United Kingdom and the United States. These countries, with the 7 largest advanced economies in the world.
- **Group of Twenty** or **G20**, an indicator variable.
  The G20 (or G-20 or Group of Twenty) is an international forum for the governments and central bank governors from Argentina, Australia, Brazil, Canada, China, France, Germany, India, Indonesia, Italy, Japan, Mexico, the Republic of Korea, the Russian Federation, Saudi Arabia, South Africa, Turkey, the United Kingdom, the United States.
- **OECD**, The Organization for Economic Co-operation and Development, an indicator variable.
  The OECD is an intergovernmental economic organization with 35 member countries, founded in 1961 to stimulate economic progress and world trade, consisting of Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Latvia, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovakia, Slovenia, South Korea, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States

- Geography
  At here, we classify all the country in geography: South America, North America, Africa, Europe, Asia (including Oceania).

- Culture and sociology
  Although there didn't exist a strict academic definition, many sociologists and historians oppose "the West and the Rest" in a categorical manner(Thompson et al., 2016).

  From a cultural and sociological aspect, "the West" is defined as including all cultures that rooted in and be derived by European cultures, it consists of Europe (European Union members), Americas (Argentina, Brazil, Canada, Chile, Colombia, Mexico, US, Uruguay) and Oceania (Australia, New Zealand).

2.2.3 Fiscal Profile

The segments in fiscal profile reflects the sustainability of a sovereign's fiscal balances. It considers:

- Fiscal performance and flexibility
- Potential risks associated with liabilities tracked by a sovereign's debt history

The first segment aims to encapsulate government budgetary flexibility. Fiscal flexibility provides governments with the ability to restore its fiscal balance after the effect of economic downturns or other shocks. On the contrary, government finance condition is subject to fiscal challenges and trends that are likely to hurt their fiscal performance. The assessment of a sovereign's revenue and expenditure flexibility is primarily qualitative.

To be more specific, we use the following factors as our determinants in analysis:

- **Fiscal Balance** To determine a sovereign's performance and flexibility, we derive an assessment based on the fiscal balance. Sometimes it also referred to as government budget balance, and we get the balance from tax revenue and the proceeds of assets sold, minus any government spending. When the balance is negative, the government has a fiscal deficit. When the balance is positive, the government has a fiscal surplus. To make the comparison easily across countries, we normalize the data, by dividing a country's GDP with the corresponding current account balance. Fiscal balance as a percentage of GDP is usually used as an instrument to measure a governments ability to meet its financing needs and to ensure good management of public finances.

  Notice that, the meaning of deficit (or surplus) may vary, when we take debt into consideration, whatever domestic or external. The deficit can be measured with or without including the payments on the debt as expenditures. We denote these two case in different ways, total deficit and primary deficit. The total deficit, which is often called the fiscal

deficit, is the primary deficit plus interest payments on the debt, i.e,

$$\begin{aligned} Total\ deficit = &\quad G_t - T_t + r * D_{t-1} \\ Primary\ deficit = &\quad G_t - T_t \end{aligned}$$

where, D_{t-1} is last year's debt (all the debt accumulated up), r is the interest rate attached to the debt, G is the government spending and T is the tax revenue for the respective year(Burda and Wyplosz, 2013). Typically, the total deficit are taken as the indicator for government budget balance, since without taking debt part into consideration, we cannot measure the ability or willingness of the government to tax its citizenry in order to service its debt.

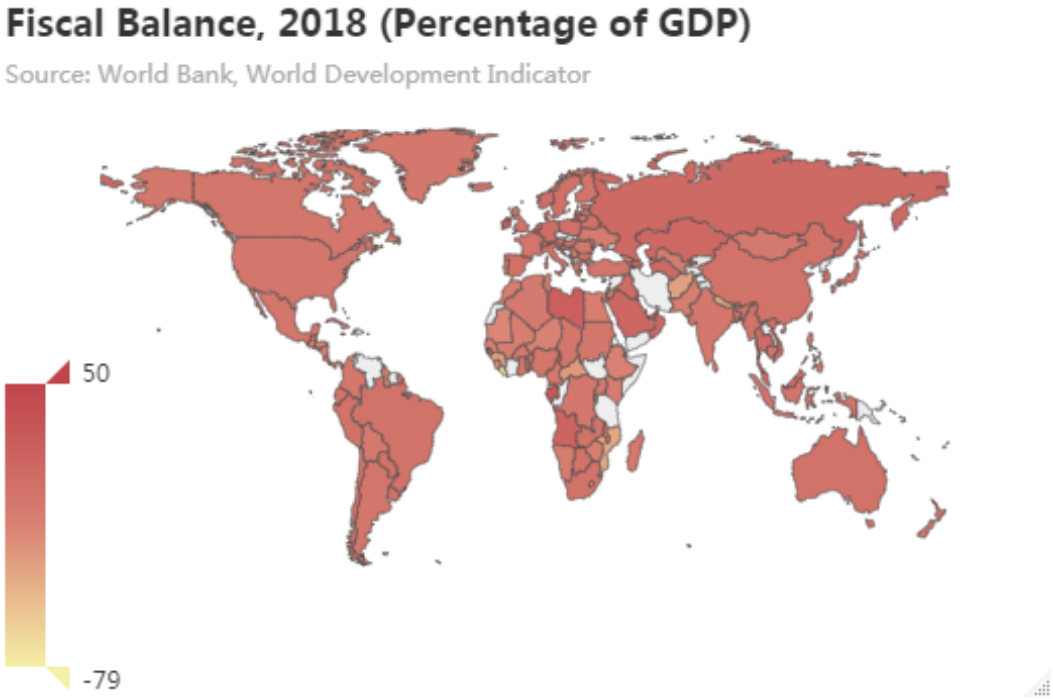


Figure 2.5: Fiscal Balance by Country, 2018

As a sample, Figure 2.5 provide a worldwide version about fiscal balance in 2018.

- **Default History** Considering other factors equal theoretically, whether a country has a default history on its issued bond or not is widely per-

ceived as a crucial signal of credit risk. Eaton (1996) introduce the importance of reputation of sovereign bond. In previous research, authors treat the default history as an indicator variable that show whether or not a certain country has experienced a sovereign bond crisis.

2.2.4 External Profile

The external profile includes factors which can reflect and measure a country's ability to meet its obligation. The term, external, refers to external economic environment, in other words, external liquidity and indebtedness. It reflects a issuer's ability and willingness to generate and obtain funds from abroad to meet public- and private-sector obligations to nonresidents.

As mentioned above, external liquidity is an indication of the economy's ability to generate the foreign exchange necessary to meet its public- and private-sector obligations to nonresidents. It seeks to capture the adequacy of official foreign exchange reserves. The quantitative indicator, which we use, is prescribed in terms of the projected ratio of the gross external financial requirement to gross reserves.

External indebtedness shows residents' assets and liabilities (in both foreign and local currency) relative to the rest of the world. This external indicator focus on public- and private-sector net external debt. It aims to capture and quantify the strength of the external balance sheet. Additionally, the external debt indicator examines potential symptoms of real exchange rate imbalance.

- **External Balance** Our key measure of external liquidity is the external balance. And in many of the existing literature which work on the determinants of sovereign ratings, they take the current account balance into consideration. Current account balance is the sum of net exports of goods and services, net primary income, and net secondary income. A current account deficit indicates that country's economics rely on funds from abroad. On the contrary, current account surplus show that a country can get rid of, in some extent, the foreign indebtedness, which is more sustainable than the countries with deficit.To be more specific, current account consists of the trade balance (the difference between the total value of exports of goods and services and the total value of imports of

goods and services), the net factor income (difference between the return on investments generated by citizens abroad and payments made to foreign investors domestically) and net cash transfers, where all these elements are measured in the domestic currency. In a mathematics way, Heakal (2014)

$$CAB = X - M + NY + NCT$$

where, X is export, M is import, NY is net factor income and NCT is net cash transfer. To make the comparison easily across countries, we typically normalize the data, by dividing a country's GDP with the corresponding current account balance.

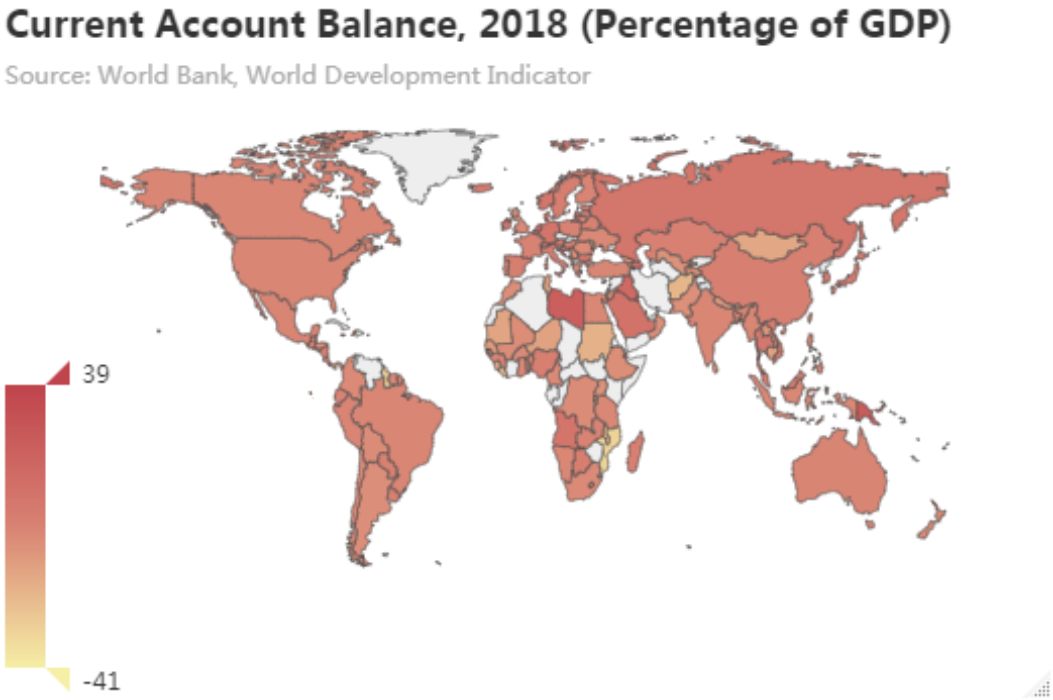


Figure 2.6: External Balance by Country, 2018

As a sample, Figure 2.6 provide a worldwide version about current account balance, percent of GDP in 2018.

- **External debt** One key measure of external indebtedness is the external debt. Gross external debt is the total amount of those actual current liabilities that require payments of principal and interest by the debtor in the future and that are owed to nonresidents by residents of an economy. A higher external debt should indicate a higher risk of default. The

weight of the debt will increases as a countrys foreign currency debt rises relative to its earnings.

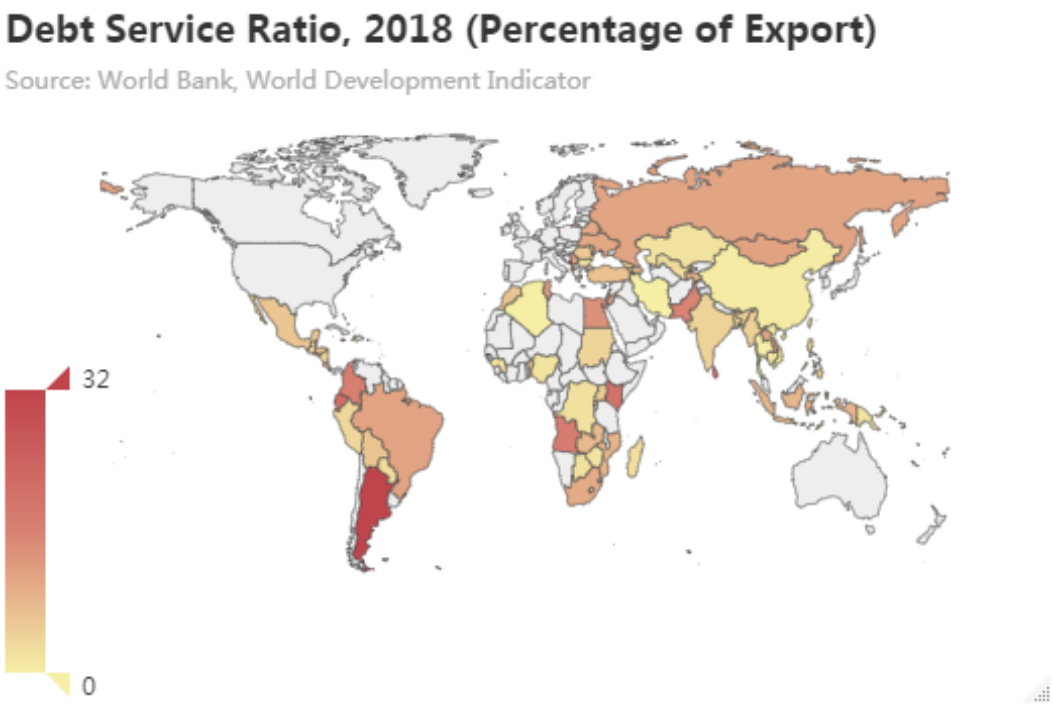


Figure 2.7: External Debt by Country, 2018

As a sample, Figure 2.7 provide a worldwide version about total external debt in 2018.

And what's more important is the willingness or ability of a country to repay the debt. A country can repay the principle and interest with its prosperous economics, while a least developed country may not repay the debt even when they are in low external debt. To make the analysis comprehensive, we introduce a new indicator. The debt service ratio is the ratio of debt service payments made by or due from a country to that country's export earnings. In other words, it's external-debt-to-export ratio. This ratio is considered to be a key indicator of a country's debt burden. (International Monetary Fund. Statistics Dept., 2014). Debt service ratio provides a quick indicator of the capability of an economy to repay external debt with enhanced revenue from sales to foreign countries. A ratio below 1 suggests that debt can be repaid rapidly, theoretically in less than one year. Conversely, the higher the ratio, the lower the country's capability to finance the debt with revenue

from exports.

2.2.5 Monetary Profile

The factor in monetary profile reflects the extent to which its monetary authority can support sustainable economic growth and attenuate major economic or financial shocks, thereby supporting sovereign creditworthiness. With the measurement of this factor, we can capture the sustainability of monetary and exchange rate policies. In reality, we can observe how the monetary factor works in the following way. The sovereign use monetary policy to deal with domestic economic stresses particularly through its control of money supply and domestic liquidity conditions. The sovereign use inflation trends as a reference of credibility of monetary policy. .

A sovereign's monetary profile results from the analysis of the credibility of its monetary policy. We use inflation rate to measure its credibility. A chief measure of the monetary policy is broad price stability, including low inflation over the economic cycle. To the extent that sovereign are assessed for monetary stability, the inflation rater is viewed as a comprehensive indicator for monetary policy efficacy, financial stability, and political effectiveness.

- **Inflation**

  Inflation is a quantitative measure of the rate which stands for a price level change of a basket of selected goods and services in an economy over a period of time. Usually expressed as a percentage, inflation indicates a decrease in the purchasing power of the currency. As inflation happens, it starts to impact the general cost of living for the common public and the monetary authority of the country. Like the central bank, the policymakers take the necessary measures to keep inflation within permissible limits and keep the economy running smoothly. In other words, A relative high inflation rate reflect, in some extent, structural problems in the government's finances.

  Inflation is measured in a variety of ways depending upon the types of goods and services considered, and when the inflation rate falls below 0 percent, it is so-called deflation. Depending upon the selected set of goods and services considered, several types of inflation values are calculated and tracked as inflation indexes. Most commonly used inflation

indexes are the Consumer Price Index (CPI) and the Wholesale Price Index (WPI).

Different from the WPI, one kind of measurement on inflation, which measures the changes in the price of goods and service in the stages before the retail level. Changes in the CPI are used to assess price changes associated with the cost of standard living, so it used as one of the most frequently statistics for identifying periods of inflation or deflation. Same with most of the previous literature, we use CPI as an indicator to measure the inflation.

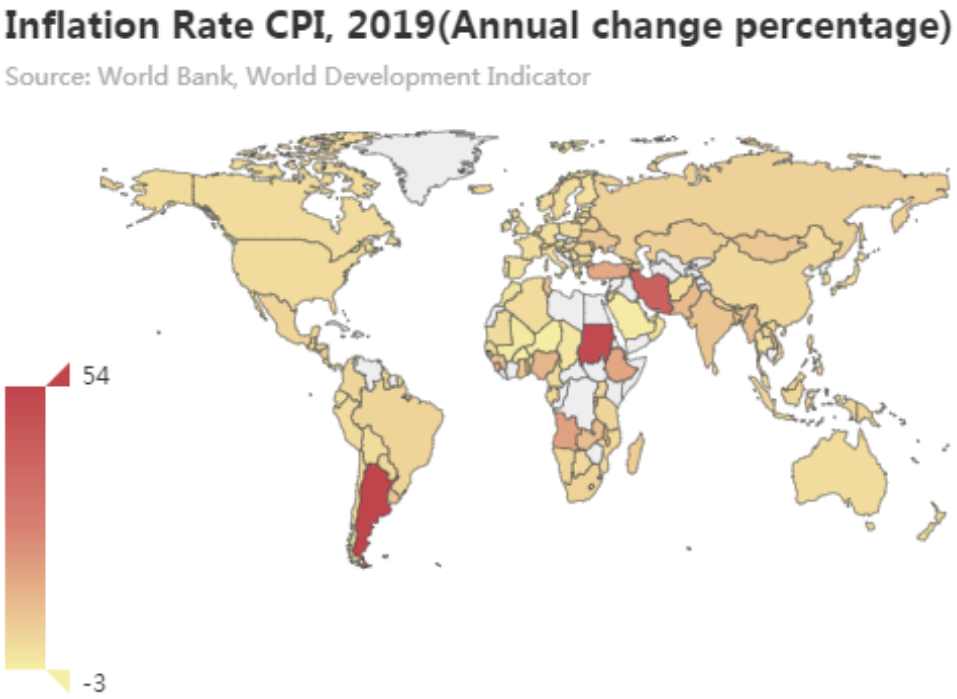


Figure 2.8: Inflation by Country, 2019

As a sample, Figure 2.8 provide a worldwide version about CPI annual change relative to 2016.

2.3 Issues in Sovereign Rating

Following the research process of sovereign rating, we organize the literature reviews in this section. Generally speaking, it contains three mainly parts: the selection of determinants, statistical methodology in analysis and rating bias in rating.

As the earliest literature concentrated on determinants of rating in academic research, Cosset and Roy (1991) focus on country risk rating which is influenced by economic and political variables. Notice, at that time, that the definition of country risk rating is a bit of different from sovereign rating, and a country risk rating is an indicator of the likelihood that a sovereign borrower will default on its debts. In the analysis, the rating score assigned to any country is a weighted average of three indicator: market indicators, credit indicators and analytical indicators, and the rating score is graded on a scale of zero to one hundred. With estimation using ordinary least squares, the evidence reveals that country risk ratings respond to some of the variables, like the level of per capita income and propensity to invest.

To examine whether the credit ratings can be explained using a set of explanatory variables, Lee (1993) estimate the problem and conclude that first, the credit ratings provide a reasonable measure of sovereign borrower's creditworthiness and second, the set of explanatory variables is significant in explaining variations in the credit ratings. The set of explanatory variables in their model, includes ratio of total foreign debt to exports, growth rate of GDP, interest rate, variability of changes in per capita GDP and inflation rate. As an improvement, they introduce additional dummy variables to represent geographical location and degree of indebtedness and the results suggests that there may be a significant group contagion in assigning credit ratings.

Cantor and Packer (1996) present the first systematic analysis of the determinants and impact of sovereign credit rating. At first step, they estimate which quantitative indicators are weighted most heavily in the determinant of rating, as a conclusion, they suggest rating assignments can be explained by a small number of well-defined criteria. While, the quantitative model they used in the paper cannot explain all variations in rating across countries, which is also stated by the rating agencies. In other words, qualitative social and political considerations are also significant determinants. To push the analysis forward, we quantify several political and social factor and take them into

consideration. The regression analysis in this paper classify rating level and corresponding determinant into different group according to the rating notch. This make the subject-specific influence absorbed into global parameters.

Afonso (2003) conduct an analysis of the possible determinants of sovereign rating by using linear, logistic and exponential transformations of the rating scales. The contribution of this paper is the improvement of quantitative model. To be more specific, they introduce non-linear transformations of rating scale instead of the linear one in existing literature. The linear transformation show some lack of accuracy for the country whose rating level located on the top end. The advantage of the use of logistic transformation make the rating change fitting reality well. when the rating level is low, ratings can rise rather quickly as the issuers deliver some improvements. Oppositely, the increase of an additional notch is slower, when the rating level is at the top end, because the requisites of sovereign quality are more demanding. All the estimated coefficients have the expected sign as the linear model estimation, what's more, the coefficients are broadly more statistically significant. Although, the paper conduct an analysis with classifying the data subject to level of development, the result is not significant, because of the lack of data. In our paper, we push this part forward, with more precise classification and bigger size of data.

In the paper of Afonso et al. (2011), the main contribution is that they employ a new specification to distinguish between short and long-run effect. In particular, they introduce the time-averages of the regressors, then they rewrite the regular regression equation into several parts: the regressor extracted by time-averages part, the time-averages regressors and the random errors. They interpret the parameter of first part, $X_i - \bar{X}_i$, as a short-term effect, and the parameter of time-averages regressor as long-term effect. In this way, they conclude what a country can do to improve its rating in the short term. As a conclusion, the paper shows that changes in GDP per capita, GDP growth rate, fiscal balance have a short-term impact on the credit rating, while the external balance and default history act on long-run terms. What's more, the author introduce a model to improve the limited dependent variable framework. In other words, they makes a continuous evaluation of a country's credit-worthiness. When evaluation stay in a value range, the country get the corresponding rating level.

Besides the existing literature from academics, there are some public sector or international organization, like OECD development centre, IMF or World Bank, working on the sovereign credit rating. Larraín et al. (1997), a pa-

per from OECD development centre, show their analysis which presents the econometric evidence on the interaction between ratings and yield spreads on sovereign bonds, and moreover, they use standard macroeconomic and financial variables, as existing literature used to determine country risk, to correct their long-term analysis for such factors. Perform Granger causality tests based on an unbalance panel data, they indicate the dollar bond spreads and a set of default determinants seem to explain somewhat well the level of credit ratings. What's more, in this paper, author state that the ratings may be characterized by the determinants of ratings, the information content of sovereign risk rating ('contaminating' rating changes with other publicly-available news), and more noteworthy, which is rarely mentioned before, the industrial organization of the rating industry. In other words, they introduce the bias problem in sovereign ratings.

From the literature above, we can see that, the methodology used which begin from Cantor and Packer (1996), that is Ordinary Least Square, has been discussed and applied in numerous research paper. Different from previous research, Bissoondoyal-Bheenick et al. (2006) introduce an ordered response model. The nature of categorical ratings data is that it is inherently ordered and discrete, and it should be argued that an ordered response model is more appropriate to the modeling of discrete and ordinal data. Besides the statistical approach, they use case-based reasoning (CBR) analysis paralleling to the ordered response model. CBR is capable of explaining and justifying its decision in the form of relevant precedents and if-then rules, and does not require a formal parametric model. As conclusion, this paper compare these two alternative techniques for the modeling of the determinants of sovereign ratings, and they both produce similar results. Apart from the technology proxy, a range of conventional macroeconomic variables are found to be significant, in particular GDP and inflation.

Besides the topics mentioned above, determinants of sovereign ratings, there are several topics discussed in sovereign rating research or industry. One of them is rating transition matrices. Rating transition matrices for sovereign are an important input to risk management of portfolios, and they are widely used both in credit portfolio management and to calculate future loss distributions for pricing purposes. Hu et al. (2002) is one of these kinds of paper. In this paper, they firstly examines which macroeconomics variables should sensibly be included in a model of sovereign rating standing, then discusses the ordered probit framework, estimation results and the transition probabilities they imply, and in the end, they introduce some Bayesian techniques

for combining different transition matrix estimates. Lando and Skødeberg (2002) also work on the estimation of rating transition matrices, and they apply continuous-time methods to estimate. Within the framework, they show to estimate the transition intensities for the Markov chain and calculate the corresponding discrete-time transition matrices.

Another important topic in sovereign rating analysis is the bias problem in rating system. Fuchs and Gehring (2017) provides the first systematic empirical assessment of claims on the concerns that revolve around the rule of the credit rating agency's "home bias", which is resulted of political economy influences and cultural distance. Their result suggest that sovereign ratings in fact exhibit biases, and the agencies assign relatively higher rating to countries to which their home-country's banks have a larger risk exposure. Moreover, they find that countries that are culturally closer receive a better rating treatment. In all, the individual analysis of all the agencies worldwide suggests that the home bias is neither only restricted to US agencies nor to the smaller agencies, but rather appears to be a generalizable phenomenon.

Chapter 3 Data Description

Throughout our analysis, we build up a rating database with sovereign bond rating data and a set of economic, financial and political dataset, which will be discussed in the following parts. For the rating database, we cover a period from 1989 to 2016 and a sample consisting of 67 countries.

3.1 Response Variable: Sovereign Ratings

Like all other bond credit ratings, the sovereign bond ratings are assessments of economic and financial situation of a certain country, moreover, reveal a measure of the ability of principal and interest repayment. While being different from other credit bonds, the sovereign bonds are issued by national government, which make sovereign ratings become more and more important. As a crucial aspect of the significance, sovereign bonds affect the stability of international financial market underlying their enormous trading volume.

While there are many credit rating agencies, the most well known are Moody's Investment Service (Moody's), Standard & Poor's (S&P) and Fitch Ratings (Fitch). Although they use different symbols to express credit risk, we can still find out counterparts in these agencies. Table 3.1 presents the rating symbols and corresponding characterization.

Our response variable is the country's sovereign rating provided by these rating agencies, Moody's, S&P and Fitch. To more specific, following the existing literature, we use sovereign's long-term foreign-currency rating. This kind of ratings is set up for government bonds which are issued in a foreign currency and have a maturity over one year. Although the demand of bonds issued in local currencies is increasing, foreign currency ratings remain the more influential and significant in the international bond market. Until the late 1980's, emerging market governments become the main part of exclusive borrowers of

Table 3.1: Moody's and S&P's Rating systems

Rating Symbols Description	Rating Level		
	Moody's	S & P	Fitch
Bonds that are judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as "gilt edged". Interest payments are protected by a large or by an exceptionally stable margin, and principal is secure.	Aaa	AAA	AAA
Bonds that are judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds. They are rated lower than the Aaa bonds because margins of protection may not be as large.	Aa1 Aa2 Aa3	AA+ AA AA-	AA+ AA AA-
Bonds that possess many favorable investment attributes and are to be considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present that suggest a susceptibility to impairment some time in the future.	A1 A2 A3	A+ A A-	A+ A A-
Bonds that are considered medium-grade obligations. Interest payments and principal security appear adequate for the present, but certain protective elements may be lacking or may be characteristically unreliable over any great length of time.	Baa1 Baa2 Baa3	BBB+ BBB BBB-	BBB+ BBB BBB-
Bonds that are judged to have speculative elements; their future cannot be considered well assured. The protection of interest and principal payments may be very moderate and thereby not well safeguarded during either good or bad times	Ba1 Ba2 Ba3	BB+ BB BB-	BB+ BB BB-
Bonds that generally lack characteristics of the desirable investment. Assurance of interest and principal payments or of maintenance of other terms of the contract over any long period of time may be small.	B1 B2 B3	B+ B B-	B+ B B-
Bonds that are of poor standing. Such issues may be in default, or there may be present elements of danger with respect to principal or interest.	Caa1 Caa2 Caa3	CCC+ CCC CCC-	CCC+ CCC CCC-
The issuer has experienced an uncured payment default on obligation, or even worse, has entered into bankruptcy filings, administration, receivership, liquidation or other formal winding-up procedure, or which has otherwise ceased business.	D	SD D	RD D

foreign currencies. This created a direct link between a balance of payment crisis and a government's default in foreign currency, and this is another reason why we focus on the sovereign's foreign-currency bond rating, which express a measurement of international financial risk. Foreign currency government bond ratings reflect agencies' opinion of the capacity and willingness of a government to mobilize foreign exchange to repay foreign currency-denominated bonds on a timely basis. (Truglia, 1999)

In our database, there consists 67 countries all through the five continents:

South America, North America, Europe, Africa and Asia (including Oceania). For a sample, the rating classifications for sovereign foreign-currency bond for each country, are presented in Table 3.2

Table 3.2: Sovereign foreign-currency long-term bond rating (June 2016)

Country	Rating Level			Country	Rating Level		
	S&P	Moody's	Fitch		S&P	Moody's	Fitch
Argentina	B-	B3	D	Lebanon	B-	B2	B
Australia	AAA	Aaa	AAA	Lithuania	A-	A3	BBB
Austria	AA+	Aa1	AA	Luxembourg	AAA	Aaa	AAA
Belgium	AA	Aa3	AA	Mexico	BBB+	A3	BBB
Bolivia	BB	Ba3	BB	Mongolia	B-	Caa1	B
Brazil	BB	Ba2	BB	Morocco	BBB-	Ba1	BBB
Bulgaria	BB+	Baa2	BBB	Netherlands	AAA	Aaa	AAA
Cameroon	B	B2	B	New Zealand	AA	Aaa	AA
Canada	AAA	Aaa	AAA	Nigeria	B	B1	BB
Chile	AA	Aa2	A+	Oman	BBB-	Baa1	BBB
China	AA-	Aa3	A	Pakistan	B	B3	NR
Colombia	BBB	Baa2	BBB	Paraguay	BB	Ba1	BB
Czech Republic	AA-	A	AA	Peru	BBB+	A3	BBB
Denmark	AAA	Aaa	AAA	Philippines	BBB	Baa2	BBB
Dominican Repub.	BB-	B1	B	Poland	BBB+	A2	A
Ecuador	B	B3	B	Portugal	BB+	Ba1	BB
Egypt	B-	B3	B	Romania	BBB-	Baa3	BBB
Estonia	AA-	A	A	Russia	BB	Ba1	BBB
Finland	AA+	Aa1	AA	Rwanda	B	B2	B
France	AA	Aa2	AA	Saudi Arabia	A-	A1	AA-
Germany	AAA	Aaa	AAA	Singapore	AAA	Aaa	AAA
Ghana	B-	B3	B	Slovenia	A	Baa3	A
Greece	B-	Caa3	C	South Africa	BBB-	Baa2	BBB
Hungary	BBB-	Baa3	BB	South Korea	AA	Aa2	AA
Iceland	BBB+	A3	BBB	Spain	BBB+	Baa2	BBB
India	BBB-	Baa3	BBB	Sri Lanka	B+	B1	BB
Indonesia	BB+	Baa3	BBB	Thailand	BBB	Baa1	BBB
Ireland	A+	A3	A	Turkey	BB	Ba1	BBB
Italy	BBB-	Baa2	BBB	Uganda	B	B2	B
Jamaica	B	B3	B	Ukraine	B-	Caa3	C
Japan	A+	A1	A	UK	AA	Aa1	AA
Kazakhstan	BBB-	Baa3	BBB	US	AA+	Aaa	AAA
Kenya	B+	B1	B	Uruguay	BBB	Baa2	BBB
Latvia	A-	A3	A				

We retrieve information about the sovereign ratings by Moody's, S&P and

Fitch via Bloomberg terminal. We downloaded the data in late April 2017 and updated in October 2017. Notice that, the dataset we obtained is an unbalanced panel, since S&P and Moody's assign ratings to different countries over varying periods of time.

For our empirical analysis, both LSDV model and LMM model, we introduce two set-up at beginning. One is a 17-point scale translation for linear regression analysis in accordance with similar approaches (begin from (Horrigan, 1966) and continuing in (Hill et al., 2010)). This translation assign the highest value of 17 for the highest rating level, "AAA" in S&P and "Aaa" in Moody's. "C" and "D" with a value of 0.

When we work on empirical analysis, we will use categorical rating level instead of the numerical ones, thus another set-up is introduced, which is refine the category as shown is Table 3.3

Table 3.3: Rating Category and Translation of Rating Scale

Interpretation	**Rating Category S&P's and Fitch**	**Moody's**	Numerical Scale
Highest quality	AAA	AAA	Aaa	16
High quality	AA	AA+	Aa1	15
		AA	Aa2	14
		AA-	Aa3	13
Strong payment capacity	A	A+	A1	12
		A	A2	11
		A-	A3	10
Adequate payment capacity	BBB	BBB+	Baa1	9
		BBB	Baa2	8
		BBB-	Baa3	7
Likely to fulfill obligations, ongoing uncertainty	BB	BB+	Ba1	6
		BB	Ba2	5
		BB-	Ba3	4
High-risk obligations	B	B+	B1	3
		B	B2	2
		B-	B3	1
Extremely high risk bond or investment	C	CCC+	Caa1	0
		CCC/CCC-	Caa2	
		CC/C	Caa3	
Defaulted	D	SD/D	C	0

3.2 Explanatory Variables

To assess the sovereign bond credit rating is not an easy problem. Intuitively speaking, we must take into consideration both the liquidity of the bond and the solvency of issuer in the market. While, Moody's and S&P provide their analytical factor of sovereign rating system (Moody's Corporation, 2001), (Standard and Poor's Corporation, 1979), where we classify in four aspects: economic performance(domestic), economic performance(external), fiscal performance and political performance, as shown in Table 3.4.

Table 3.4: Sovereign Rating Analytical factors

	Domestic Performance	External Performance	Fiscal Performance	Political Performance
S&P	- Structure of national economies - Long-run economic growth - Short-Run business cycle	-External liquidity -External balance	-Debt burden -Budget balance	-Political risks -International co-operation
Moody's	- National economic size - Long-Run trends - GDP per capita	- External risk (debt) - External balance	- Government Budget balance	-Governance -International co-operation

To deal with problem that how to measure these different kinds of performance, we build on and employ the sets of variables provided by the existing literatures: Haque et al. (1996) and Cantor and Packer (1996) provide the rationale for several factors which measure macroeconomic and financial performance, such as, per capita income, GDP growth, inflation, economic balance. Ul Haque et al. (1998) show the relative importance of political factors. Archer et al. (2007) and Hill et al. (2010) introduce an important property of the factors, which is country-specific, and causal relationship between factors and ability of a country to repay the issued bonds. For more detail in criteria definition, we check the handbook by Moody's Corporation (2001).

We identify the factors mentioned above as a set of quantitative and qualitative variables, which we can reorganize in four main aspects:

- **Economic Variables**: GDP per capita, Real GDP, GDP Growth
- **Fiscal Variables**: Fiscal balance, Economic development

- **External Variables**: External balance, Debt service ratio
- **Monetary Variables**: Inflation
- **Other Variables**: Default history, Regional indicators, Political indicators

While, we will provide description of each variables in the following. Introducing brief economic definition, we use the data description from World Development Indicators database of World Bank as reference.

3.2.1 Economic Variables

- **GDP per capita**

  As a full-information terms, **World Bank GDP per Capita PPP Current Intl Dollar** is represented by GDP per capita. PPP here means purchasing power parity. Gross domestic product converted to international dollars using purchasing power parity rates. An international dollar has the same purchasing power over GDP as the U.S. dollar has in the United States. Then, the GDP per capita of a certain country is comparable with others underlying a uniqle measurable basis, an international dollar. GDP at purchaser's prices is the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of the products.

  We get the data from World Bank, International Comparison Programme database. Data updated with a 1-2 year lag due to the large amount of data processed by the World Bank. Yearly data is therefore available around late November of the current year for the previous year.

- **Real GDP**

  As a full-information terms, **World Bank GDP Constant 2010 Prices USD** is represented by Real GDP. GDP is the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources. Data are in constant 2010 U.S. dollars. Dollar figures for GDP are converted from

domestic currencies using single-year official exchange rates. For a few countries where official exchange rate does not reflect the rate effectively applied to actual foreign exchange rates. For a few countries where the official exchange rate does not reflect the rate effectively applied to actual foreign exchange transactions, an alternative conversion factor is used.

We get the data from World Bank national accounts data, and OECD National Accounts data files. Data is updated with a 1-2 year lag due to the large amount of data processed by the World Bank. Yearly data is therefore available around September of the current year for the previous year.

- **GDP per capita Growth**

  As a full-information terms, **World Bank GDP per capita Growth in Annual Change** is represented by GDP per capita Growth. Annual percentage growth rate of GDP per capita based on constant local currency. GDP per capita is gross domestic product divided by midyear population. GDP at purchaser's prices is the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources.

  We get the data from World Bank national accounts data, and OECD National Accounts data files. Data is updated with a 1-2 year lag due to the large amount of data processed by the World Bank. Yearly data is therefore available around September of the current year for the previous year.

- **Inflation**

  As a full-information terms, **World Bank Consumer Price Index (2010 Base 100) in Annual Change** is represented by Inflation. Consumer price index is measurement about the cost of a certain fixed set, or a fixed "basket", of consumer goods and services. However, the basket of goods and services priced by CPI is not literally same over different period, and is periodically changed to reflect the different mix of consumer goods and services at different period. In other words, consumer price index reflects changes in the cost to the average consumer of acquiring a fixed basket of goods and services that may be fixed or changed at specified intervals, such as yearly. Moreover, the annual inflation we

used here refers to the percent change of the CPI compared to the same month of the previous year.

We get the data from International Monetary Fund, International Financial Statistics and data files. Data is updated with a 1-2 year lag due to the large amount of data processed by the World Bank. Yearly data is therefore available around September of the current year for the previous year.

3.2.2 Financial Variables

- **Fiscal Balance**

  At here, we express fiscal balance in another way which provide more information about the factor. That is the percentage of government budget balance in GDP, normalizing the data, by dividing the budget balance by GDP, which enables easy comparisons across countries and indicates whether a national government saves or borrows money. The crucial part of this factor is government budget balance, which records the difference between national government revenues and expenditures. If the difference is a positive number, it means revenues exceeded expenditures, and we call this situation as budget surplus, otherwise, it is budget deficit. Intuitively speaking, countries with high budget deficits generally have more difficulty raising funds to finance expenditures, furthermore to repay the issued bonds, than those with lower deficits.

  We use total deficit (or surplus) as government budget balance, since without debt payment part, we can't measure the ability or willingness of a government to tax its citizenry to cover current expenses and service its debt. We get the dataset from World Bank national accounts data and International Monetary Fund via Bloomberg.

- **External Balance** At here, we express external balance in full information way. That is the percentage of current account balance in GDP, normalizing the data, by dividing the current account balance by GDP, which enables easy comparisons across countries and indicates whether a national government is a net lender to the rest of the world or net borrower. In other words, the ratio of the current account balance to the GDP provides an indication of the countrys level of international competitiveness. The current account tracks all transactions, excluding

financial transactions, that involve economic values. Major components include trade in goods, trade in services, income and current transfers.

A current account deficit is symbol of an economy that is a net borrower to the rest of the world. It shows how much a country is investment beyond the saving. What this means is that the country is using resources from rest of the world to meet its domestic consumption and investment requirements and heavily relying on funds from aboard, finally it make the country's growth under foreign indebtedness and unsustainable over time. A current account surplus can be analyzed in the opposite way.

We get the data from International Monetary Fund, Balance of Payments Statistics Yearbook and data files, and World Bank and OECD GDP estimates.

- **Debt service ratio** The debt service ratio is the ratio of debt service payments made by or due from a country to that countrys export earnings. I.e, it's external-debt-to-export ratio. This ratio is considered to be a key indicator of a countrys debt burden.(International Monetary Fund. Statistics Dept., 2014).

  We can see this indicator consists two components: gross external debt and exports, and now consider these two parts in the ratio. Gross external debt is the outstanding amount of those actual current liabilities that require payments of interest and principal by the debtor. These debts are owed by residents of an economy to nonresidents. Data on external debt are gathered through the World Bank's Debtor Reporting System (DRS). Exports measure the movement of merchandise trade leaving a country. This measure tracks the value of merchandise trade. We get the data of export from International Monetary Fund, using the Direction of Trade Statistics (DOTS), a product of International Monetary Fund , which presents figures on the value of merchandise exports and imports by trade partners as well as area and world aggregates showing trade flows between major areas of the world. All figures are presented in U.S. dollars.

3.2.3 Other Variables

- **Default History** In our analysis, we use the default history as an indicator variable that show whether or not a certain country has experienced

a sovereign bond crisis. A country has a default history on its issued bond or not is widely perceived as a crucial signal of credit risk. In our analysis, the indicator variable equals to 1, if a country has defaulted on its bond since 1983. We get these part of data sources from International Monetary Fund.

- **Economic Development** Besides the GDP per capita, which can measure the level of development, the rating agencies also use other factors to reflect the relationship between development and risk. In our study, we use two different indicator variables to measure the causal relationship, which make the analysis more comprehensive. One is industrialized classification, another is Least Developed Countries (LDCs) classification, both of these two are provided by International Monetary Fund.
- **Political Factor** At this part, we use several variables to test for the presence of relationship between rating and economic cooperation, geopolitical or culture affects. In the following, we classify the variables into three categories: economic cooperation, politics, sociology.

Economic Cooperation

  - **Group of Seven** or **G7**, an indicator variable for membership of G7.
    The Group of Seven or G7 is a group consisting of Canada, France, Germany, Italy, Japan, the United Kingdom and the United States. These countries, with the 7 largest advanced economies in the world. If a country is the member of G7, the indicator variable takes 1, and if not, the indicator takes 0.
  - **Group of Twenty** or **G20**, an indicator variable for membership of G20.
    The G20 (or G-20 or Group of Twenty) is an international forum for the governments and central bank governors from Argentina, Australia, Brazil, Canada, China, France, Germany, India, Indonesia, Italy, Japan, Mexico, the Republic of Korea, the Russian Federation, Saudi Arabia, South Africa, Turkey, the United Kingdom, the United States. If a country is the member of G20, the indicator variable takes 1, and if not, the indicator takes 0.
  - **OECD**, The Organization for Economic Co-operation and Development, an indicator variable for membership of OECD.

The OECD is an intergovernmental economic organization with 35 member countries, founded in 1961 to stimulate economic progress and world trade, consisting of Australia, Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Latvia, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovakia, Slovenia, South Korea, Spain, Sweden, Switzerland, Turkey, United Kingdom, United States. If a country is the member of OECD, the indicator variable takes 1, and if not, the indicator takes 0.

Culture and sociology

Although there didn't exist a strict international definition, many sociologists and historians oppose "the West and the Rest" in a categorical manner.(Thompson et al., 2016) From a cultural and sociological aspect,"the West" is defined as including all cultures that rooted in and be derived by European cultures, it consists of Europe (European Union members), Americas (Argentina, Brazil, Canada, Chile, Colombia, Mexico, US, Uruguay) and Oceania (Australia, New Zealand). If a country is in the list of "the West", the indicator variable takes 1, and if not, the indicator takes 0.

Chapter 4 Statistical Method

In the beginning of this methodology section, we briefly introduce a linear model. This general model is used in most of the previous literature and research which focus on the determines of sovereign rating. Notice that, in the existing literature, the researches assume homogeneity across the countries.

In the following parts, we introduces our analysis on the longitudinal data of sovereign rating. Here, We carry out a regression problem with the longitudinal data, modeling the heterogeneity by using two different methods: fixed parameters and random quantities. The fixed parameters that account for the heterogeneity are known as *fixed effect*. In contrast to fixed effect, the random quantities which are used to model heterogeneity in lieu of fixed parameters are known as *random effect*, and the analysis is cast in the mixed linear model framework.

Section 4.2 discusses the fixed effect model, which treats the individual parameters as unknown fixed value to be estimated. Section 4.3 assumes the individual parameters which are under the same normal distribution. Section 4.4 discusses the feedback effect analysis, at here, we introduce indicator variables to capture the rating history movement, so as to analyze the effect of which on the rating assignment.

To clarify the idea above, in this section we concentrate the following model, with same notations through this dissertation,

$$y_i = X_i'\beta + D_i\alpha_i + \epsilon_i \qquad i = 1, \ldots, n \tag{4.1}$$

with $y_i^{T_i \times 1} = (y_{i1}, \ldots, y_{iT_i})'$, $X_i^{k \times T_i} = (X_{i1}^{k \times 1}, \ldots, X_{iT_i}^{k \times 1})$, $\epsilon_i^{T_i \times 1} = (\epsilon_{i1}, \ldots, \epsilon_{iT_i})'$, and $D_i^{T_i \times 1} = (1, \ldots, 1)'$. While, y_{it_i} is the response variable, in our study which is the sovereign rating, for the ith country at time t, $x_{it}^{k \times 1}$ contains the explanatory variables, $\beta^{k \times 1}$ is the coefficient to be estimated and α_i is the heterogeneity parameter, which vary by subject.

Under cross-sectional regression model, $y_{it} = \alpha_i + X\beta + \epsilon_{it}$ at a certain period, the disturbance term ϵ_{it} includes the information in α_i. And, the parameter α_i are non-estimated without repeated observations. As a certain t, we have $n + K$ parameters to be estimated, meanwhile, we only get n observations.

Comparing to cross-sectional regression model, longitudinal data models provide a advantage, in another word, ability to separate the subject-specific parameter α_i from the error terms ϵ_{it}. With the help of longitudinal data models, our estimations of parameter effects, both the global parameter β and the subject-specific parameter α_i, become more precise. And taking into consideration of heterogeneity, we can get more inferences about subject-specific effect in sovereign rating problems.

Besides investigating determines of sovereign rating and heterogeneous effect in rating assignment, we introduce a set of indicator to represent the rating history changes, which are named as feedback effect. We denote two variables, one-period feedback, which show the rating movement comparing current level and previous one, and two-period lagged feedback, which compares the previous level and two-period preceding ones. With OLS estimation, we can estimate the parameters of the feedback effect indicator.

4.1 General Linear Model

As discussed in Section Two and Three, the explanatory variables includes economic, financial and geopolitical variables. And as the first step, we assume the homogeneity in sovereign rating across the countries, i.e, there exists a constant term.

The general linear model to estimate is as follows:

$$\begin{aligned} y_{i,t} =& \alpha + \beta_1 GDP_{i,t} + \beta_2 GDP_{i,t} + \beta_3 GDPgrowth_{i,t} + \beta_4 INF_{i,t} + \beta_5 FisBal_{i,t} \\ &+ \beta_6 ExtBal_{i,t} + \beta_7 DebtX_{i,t} + \beta_8 DEF_{i,t} + \beta_9 DEVELOP_{i,t} + \epsilon_{i,t} \end{aligned} \tag{4.2}$$

where, y_i is the rating level for country i. And we have the following ex-

planatory variables: per capita GDP, real GDP, GDP growth, inflation, fiscal balance, external balance, debt service ratio, default history and development indicator.

As a further study, we take several geopolitical indicator variable into consideration. Setting all of these explanatory variable as a vector, we can rewrite the model 4.3 as:

$$y_{i,t} = \alpha + X'_{i,t}\beta + \epsilon_{i,t} \tag{4.3}$$

with X_i contains the explanatory variables, and β is a vector of coefficient to be estimated, α is the homogeneity parameter.

To fit a regression model 4.3 to the observed data, the least squares method choose a, b, where $b = (b_1, b_2, \cdots, b_k)'$ to minimized the sum of squared residuals (SSR),

$$SSR = \sum_{i=1}^{n}\sum_{t=1}^{T_i}(y_{it} - (a + x'_{it}b))^2 \tag{4.4}$$

It is convenience to write a as $ax0_{it}$, where $x0 = 1$. Then the regression model 4.3 can be written in matrix form as

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{4.5}$$

where

$$\boldsymbol{Y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} \alpha \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix}, \quad \boldsymbol{\epsilon} = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{pmatrix}, \quad \boldsymbol{X} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix}$$

with $y_i^{T_i\times 1} = (y_{i1}, \ldots, y_{iT_i})'$, $X_i^{T_i\times(k+1)} = (X_{i1}^{(k+1)\times 1}, \ldots, X_{iT_i}^{(k+1)\times 1})'$, $\epsilon_i^{T_i\times 1} = (\epsilon_{i1}, \ldots, \epsilon_{iT_i})'$. And $X_{i,T_i}^{(k+1)\times 1} = (1, x_{1,i,T_i}, \cdots, x_{1,k,T_i})$

The vector of least squares estimates of the β is given by,

$$\hat{\beta_{OLS}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y} \tag{4.6}$$

Then we can estimate α by

$$\hat{\alpha}_{OLS} = \bar{\boldsymbol{Y}} - \bar{\boldsymbol{X}}'\hat{\beta}_{OLS} \tag{4.7}$$

4.2 Fixed Effect Model

Here, using fixed parameters to represent the heterogeneity, we carry out a regression problem under longitudinal data model. There is nonrandom quantities for the heterogeneity which is known as fixed effect.

As mentioned above, there is a drawback in cross-sectional regression model, in other words, we cannot estimate the coefficient for time-invariant variable. The dataset we analyzed in this dissertation is a longitudinal panel dataset, which is suitable for fixed effect approach analyzing the effect of time-varying and time-invariant covariates separately (Greene, 2011).

Following the general setting and the basic model, equation 4.1. We estimate the fixed effect model in the following, which is written in matrix form,

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \beta + \begin{bmatrix} D_1 & 0 & \cdots & 0 \\ 0 & D_2 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & D_n \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{bmatrix} \tag{4.8}$$

where $y_i^{T_i \times 1} = (y_{i1}, \ldots, y_{iT_i})'$, $X_i^{T_i \times k} = (X_{i1}^{k \times 1}, \ldots, X_{iT_i}^{k \times 1})'$, $\epsilon_i^{T_i \times 1} = (\epsilon_{i1}, \ldots, \epsilon_{iT_i})'$, and $D_i^{T_i \times 1} = (1, \ldots, 1)'$.

Equation 4.8 is known as the "least square dummy variable" (LSDV) model, since the observed values of coefficient α_i take the form of dummy

variables (Hsiao, 2003). Rewrite as a reduced form,

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{D\alpha} + \boldsymbol{\epsilon} \tag{4.9}$$

4.2.1 Fixed Effect Model: Ordinary Least Squares Estimation

Returning to the model, $y_i = \alpha_i + X_i'\beta + \epsilon_i$, we now estimate the coefficient of global and subject-specific parameter, β and α_i. Denote a and b, where $b = (b_1, b_2, \cdots, b_k)'$, as the 'possible' value of coefficient, we have the sum of squares in the model, as

$$SS(a,b) = \sum_{i=1}^{n}\sum_{t=1}^{T_i}(y_{it} - (a_i + x_{it}'b))^2 \tag{4.10}$$

With the help of $\frac{\partial}{\partial a}SS(a,b) = 0$ and $a = \bar{y_i} - \bar{x_i}'b$, we get the OLS estimators,

$$\hat{\beta}_{OLS} = \big(\sum_{i=1}^{n}\sum_{t=1}^{T_i}(x_{it} - \bar{x_i})^2\big)^{-1}\sum_{i=1}^{n}\sum_{t=1}^{T_i}(x_{it} - \bar{x_i})(y_{it} - \bar{y_i})$$

and

$$\hat{\alpha}_{iOLS} = \bar{y_i} - \bar{x_i}'\hat{\beta}_{OLS} \tag{4.11}$$

where $\bar{y_i} = (\sum_{t=1}^{T_i} y_{it})/T_i$ and $\bar{x_i} = (\sum_{t=1}^{T_i} x_{it})/T_i$.

4.2.2 Fixed Effect Model: Generalized Least Squares Estimation

Turning to generalized linear square estimation, we relax the assumption of error term, ϵ. In other words, for a full set of observations, there exists a variance-covariance matrix, denoted as Ω, instead of the identity matrix in ordinary linear model. And the matrix can be written as $\Omega = cov(\epsilon_s, \epsilon_t)$.

Consider linear longitudinal data with regression model for each individual at single period,

$$y_{it} = X_{it}'\beta + \alpha_i + \epsilon_{it} \qquad i = 1, \ldots, n; \quad t = 1, \ldots, T_i$$

Then we introduce another equation,

$$\bar{y_i} = \bar{X_i}'\beta + \alpha_i + \bar{\epsilon_i}$$

where $\bar{y_i} = \frac{1}{T_i}\sum_{t=1}^{T_i} y_{it}$, $\bar{X_i} = \frac{1}{T_i}\sum_{t=1}^{T_i} X_{it}$ and $\bar{\epsilon_i} = \frac{1}{T_i}\sum_{t=1}^{T_i} \epsilon_{it}$, consequently we have

$$y_{it} - \bar{y_i} = (X_{it} - \bar{X_i})'\beta + (\epsilon_{it} - \bar{\epsilon_i}) \tag{4.12}$$

From 4.12, we can get a GLS estimation of β, which are shown as follows with notation, $\widetilde{\epsilon_{it}} = \epsilon_{it} - \bar{\epsilon_i}$:

Note that,

$$\begin{aligned} Var(\widetilde{\epsilon_{it}}) &= Var\big[(1-\frac{1}{T_i})\epsilon_{it} - \sum_{r\neq t}\frac{1}{T_i}\epsilon_{ir}\big] = \frac{T_i-1}{T_i}\sigma^2 \\ Cov(\widetilde{\epsilon_{it}}, \widetilde{\epsilon_{is}}) &= Cov\big[(1-\frac{1}{T_i})\epsilon_{it} - \sum_{r\neq t}\frac{1}{T_i}\epsilon_{ir}, (1-\frac{1}{T_i})\epsilon_{is} - \sum_{r\neq s}\frac{1}{T_i}\epsilon_{is}\big] \\ &= -2(1-\frac{1}{T_i})\frac{1}{T_i}\sigma^2 \end{aligned} \tag{4.13}$$

Hence,

$$Cov(\tilde{\epsilon_{\mathbf{i}}}) = \Big[\frac{(T_i+2)(T_i-1)}{T_i^2} I_{T_i} - \frac{2(T_i-1)}{T_i^2} J_{T_i}\Big]\sigma^2 \tag{4.14}$$

where I_{T_i} is a $T_i \times T_i$ identity matrix and J_{T_i} is a $T_i \times T_i$ matrix with all elements equals 1. What's more, at here, we denote the covariance matrix as

V_i

The generalized least-squares sum of squares is

$$SS(b) = \sum_{i=1}^{n}\sum_{i=1}^{T_i}\big((y_{it}-\bar{y}_i)-(X_{it}-\bar{X}_i)'b\big)'V_i^{-1}\big((y_{it}-\bar{y}_i)-(X_{it}-\bar{X}_i)'b\big) \quad (4.15)$$

b here is estimator of β, where $b = (b_1, b_2, \cdots, b_k)'$.

Following the same method as used in previous section, we take partial derivative with respect to b and setting to zero,

$$\frac{\partial}{\partial b}SS(b) = -2\sum_{i=1}^{n}\sum_{i=1}^{T_i}(X_{it}-\bar{X}_i)V_i^{-1}\big((y_{it}-\bar{y}_i)-(X_{it}-\bar{X}_i)'b\big) \quad (4.16)$$

then we get

$$\hat{\beta}_{GLS} = \Big(\sum_{i=1}^{n}\sum_{i=1}^{T_i}(X_{it}-\bar{X}_i)V_i^{-1}(X_{it}-\bar{X}_i)'\Big)^{-1}\sum_{i=1}^{n}(X_{it}-\bar{X}_i)V_i^{-1}(y_{it}-\bar{y}_i) \quad (4.17)$$

Then we can estimate α_i by

$$\hat{\alpha}_{i,GLS} = \bar{y}_i - \bar{X}_i{}'\hat{\beta}_{GLS} \quad (4.18)$$

Consequently, we can get the fitted value,

$$\hat{y}_{it} = X'_{it}\hat{\beta} + \hat{\alpha}_i = \bar{y}_i + \big(X_{it}-\bar{X}_i\big)'\hat{\beta} \quad (4.19)$$

and residuals,

$$\begin{aligned} e_{it} &\triangleq y_{it} - \hat{y}_{it} = (y_{it} - \bar{y}_i) - \big(X_{it} - \bar{X}_i\big)'\hat{\beta} \\ &= \big(X_{it} - \bar{X}_i\big)'\big(\beta - \hat{\beta}\big) + (\epsilon_{it} - \bar{\epsilon}_i) \end{aligned} \tag{4.20}$$

4.2.3 Fixed Effect Model: Multivariable with GLS estimation

In previous section, we introduce a subject-specific intercept, which vary by individuals, in the model. To make it widely used, we set up a more compact model with multivariables. At here, we can treat the fixed effect parameter as slopes which vary by subject.

Consider linear longitudinal data model with regression model,

$$y_i = X_i'\beta + Z_i\alpha_i + \epsilon_i \qquad i = 1, \ldots, n$$

where $y_i^{T_i \times 1} = (y_{i1}, \ldots, y_{iT_i})'$, $\epsilon_i^{T_i \times 1} = (\epsilon_{i1}, \ldots, \epsilon_{iT_i})'$, the explanatory variables $Z_i = (Z_{it,1}, \ldots, Z_{it,k})$ and what's more, the variance-covariance matrix of ϵ_i, Ω_i, is assumed known.

The generalized least-squares sum of squares is

$$SS(a, b) = \sum_{i=1}^{n} \big(y_i - (X_i'b + Z_i a_i)\big)'\Omega_i^{-1}\big(y_i - (X_i'b + Z_i a_i)\big) \tag{4.21}$$

a, b here are estimators of α and β, where $\alpha = (\alpha_1, \cdots, \alpha_n)'$ and $b = (b_1, b_2, \cdots, b_k)'$.

Taking partial derivatives of SS with respect to each subject-specific coefficient, α_i, and set them equal to zero,

$$\frac{\partial}{\partial a_i} SS(a, b) = -2Z_i'\Omega_i^{-1}\big(y_i - (X_i'b + Z_i a_i)\big) \tag{4.22}$$

then,

$$a_i = (Z_i'\Omega_i^{-1}Z_i)^{-1}Z_i'\Omega_i^{-1}(y_i - X_i b) \tag{4.23}$$

Furthermore, we introduce a matrix, R_i, where $R_i = \Omega_i^{-\frac{1}{2}}Z_i(Z_i'\Omega_i^{-1}Z_i)^{-1}Z_i'\Omega_i^{-\frac{1}{2}}$. Then, we can see that $R_i \cdot R_i = R_i$, which means idempotent, what's more, R_i is symmetric.

We can rewrite equation (4.23) as $\Omega_i^{-\frac{1}{2}}Z_i a_i = R_i\Omega_i^{-\frac{1}{2}}(y_i - X_i'b)$

$$\begin{aligned}
\Omega_i^{-\frac{1}{2}}\big(y_i - (X_i'b + Z_i a_i)\big) &= \Omega_i^{-\frac{1}{2}}(y_i - X_i'b) - \Omega_i^{-\frac{1}{2}}Z_i\alpha_i \\
&= \Omega_i^{-\frac{1}{2}}(y_i - X_i'b) - R_i\Omega_i^{-\frac{1}{2}}(y_i - X_i'b) \\
&= (I - R_i)\Omega_i^{-\frac{1}{2}}(y_i - X_i'b) \\
&= Q_i\Omega_i^{-\frac{1}{2}}(y_i - X_i'b)
\end{aligned} \tag{4.24}$$

Since R_i is idempotent and symmetric, $Q_i = I - R_i$ is also idempotent and symmetric.

Then we have,

$$SS(a,b) = \sum_{i=1}^{n}(y_i - X_i'b)'\Omega_i^{-\frac{1}{2}}Q_i\Omega_i^{-\frac{1}{2}}(y_i - X_i'b) \tag{4.25}$$

Taking partial derivative with respect to b and setting to zero,

$$\hat{\beta} = (\sum_{i=1}^{n} X_i'\Omega_i^{-\frac{1}{2}}Q_i\Omega_i^{-\frac{1}{2}}X_i)^{-1}\sum_{i=1}^{n} X_i'\Omega_i^{-\frac{1}{2}}Q_i\Omega_i^{-\frac{1}{2}}y_i \tag{4.26}$$

and

$$\hat{\alpha}_i = (Z_i'\Omega_i^{-1}Z_i)^{-1}Z_i'\Omega_i^{-1}(y_i - X_i'\hat{\beta}) \tag{4.27}$$

4.3 Random Effect Model

In contrast to the character of model in previous section, in which the heterogeneity is modeled using fixed parameters, this section introduces a different model framework, and at here the heterogeneity is featured by random quantities. This model is called error-component model since the variation of model comes from two aspects: the random effect, α_i, and, the random error, ϵ_{it}. Random effect model, here, assumes $\alpha_i \overset{i.i.d}{\sim} N(0, \sigma_\alpha^2)$ to be the unknown subject-specific random effect. At here, we assume that α_i, ϵ_{it} are independent.

The random effect model, which we analyze in the dissertation, is shown as follows:

$$y_{it} = X_{it}'\beta + \alpha_i + \epsilon_{it} \qquad i = 1, \ldots, n;\ t = 1, \ldots, T_i$$

With assumptions: (I) $\alpha_i \overset{i.i.d}{\sim} N(0, \sigma_\alpha^2)$ and (II) $\{\alpha_i\}$, $\{\epsilon_{it}\}$ are independent. Then rewrite the model for individual i:

$$Y_i = X_i\beta + D_i\alpha_i + \epsilon_i \qquad i = 1, \ldots, n$$

where $Y_i = (y_{i1}, \ldots, y_{iT_i})$, $X_i = (X_{i1}, \ldots X_{iT_i})'$, $\epsilon_i = (\epsilon_{i1}, \ldots, \epsilon_{iT_i})'$ and $D_i = (1, \ldots, 1)'$.

Since the variation of the model is different from the fixed effect one, we calculate the variance of response variable first, which we denote as V_i for $Var(Y_i)$:

$$\begin{aligned} E(Y_i|\alpha_i) &= X_i\beta + D_i\alpha_i \\ E(Y_i) &= E(E(Y_i|\alpha_i)) = X_i\beta \\ Var(Y_i|\alpha_i) &= E\big[(Y_i|\alpha_i - E(Y_i|\alpha_i))(Y_i|\alpha_i - E(Y_i|\alpha_i))'\big] = E(\epsilon_i\epsilon_i') \\ &= \sigma^2 I_{T_i} \end{aligned}$$

Then, we get,

$$\begin{aligned} V_i &= Var(E(Y_i|\alpha_i)) + E(Var(Y_i|\alpha_i)) \\ &= Var(X_i\beta + D_i\alpha_i) + E(\sigma^2 I_{T_i}) = E(\alpha_i^2 J_{T_i}) + \sigma^2 I_{T_i} \\ &= \sigma_\alpha^2 J_{T_i} + \sigma^2 I_{T_i} \end{aligned} \tag{4.28}$$

where I_{T_i} is a $T_i \times T_i$ identity matrix and J_{T_i} is a $T_i \times T_i$ matrix with all elements equals 1. According to the Sherman-Morrison formula,

$$V_i^{-1} = \frac{1}{\sigma^2}\big(I_{T_i} - \frac{\sigma_\alpha^2}{\sigma^2 + \sigma_\alpha^2 T_i} J_{T_i}\big) = \frac{1}{\sigma^2}\big(I_{T_i} - \frac{\tau}{1 + T_i\tau} J_{T_i}\big) \tag{4.29}$$

where $\tau = \frac{\sigma_\alpha^2}{\sigma^2}$.

Since

$$\begin{aligned} Y_i|\alpha_i &\sim N(X_i\beta + D_i\alpha_i, \sigma^2 I_{T_i}), \\ \alpha_i &\sim N(0, \sigma_\alpha^2), \end{aligned}$$

we can get the probability density functions of Y_i by integrating the random effects, α_i, from $f(Y_i) = \int_{-\infty}^{\infty} f(Y_i|\alpha_i) f(\alpha_i) d\alpha_i$, which can be rewrite as $f(Y_i) = \frac{1}{\sqrt{(2\pi)^{T_i} det(V_i)}} \exp\Big(-\frac{1}{2}(Y_i - X_i\beta)' V_i^{-1} (Y_i - X_i\beta)\Big)$. Thus we have

$$Y_i \sim N(X_i\beta, V_i)$$

Furthermore, we can get the closed form of full likelihood function as: $L(X, Y; \beta, \sigma^2, \sigma_\alpha^2) = \prod_{i=1}^{n} L_i(X_i, Y_i; \beta, \sigma^2, \sigma_\alpha^2)$, and the log-likelihood function for each subject i can be write as:

$$l_i(\beta, \sigma^2, \sigma_\alpha^2) = -\frac{1}{2}\Big\{T_i \log(2\pi) + \log\det(V_i) + (Y_i - X_i\beta)' V_i^{-1} (Y_i - X_i\beta)\Big\} \tag{4.30}$$

where $V_i = \sigma^2 I_{T_i} + \sigma_\alpha^2 J_{T_i}$.

From Demidenko (2013), we have:

$$\log\det(\sigma^2 I_{T_i} + \sigma_\alpha^2 J_{T_i}) = T_i \log\sigma^2 + \log(1 + \frac{T_i\sigma_\alpha^2}{\sigma^2}) \tag{4.31}$$

Given $\sigma^2, \sigma_\alpha^2$, the best linear unbiased estimator of the global parameter is the generalized least square estimator, which we denote as $\hat{\beta}_{GLS}$, given by

$$\begin{aligned}\hat{\beta}_{GLS} &= \Big(\sum_{i=1}^{n} X_i' V_i^{-1} X_i\Big)^{-1} \sum_{i=1}^{n} X_i' V_i^{-1} Y_i \\ &= \Big(\sum_{i=1}^{n} X_i'\big(I_{T_i} - \frac{\tau}{1+T_i\tau} J_{T_i}\big) X_i\Big)^{-1} \sum_{i=1}^{n} X_i'\big(I_{T_i} - \frac{\tau}{1+T_i\tau} J_{T_i}\big) Y_i\end{aligned} \tag{4.32}$$

which can be easily shown when we maximize the log-likelihood function (4.30) with given $\sigma^2, \sigma_\alpha^2$.

The consequential step is to estimate σ^2 and σ_α^2. When analyzing the linear regression models constructed by quantitative dependent variables, there is a commonly used methodology, *demeaning variable*. The within-subject means for each variable (both the explanatory variables, $\mathbf{X}$, and the response variables, $\mathbf{Y}$) are subtracted from the observed variables. This basically gets rid of all subject-specific variability (which may be contaminated by omitted variable bias) and leaves only the within-subject variability to analyze.

To estimate the variance of error term, we analyze the model for each individual subject i, then we have $y_{it} = X_{it}'\beta + \alpha_i + \epsilon_{it}$ and $\bar{y}_i = \bar{X}_i{}'\beta + \alpha_i + \bar{\epsilon}_i$, where $\bar{y}_i, \bar{X}_i$ and $\bar{\epsilon}_i$ are the demeaning variable, which are given by $\frac{1}{T_i}\sum_{t=1}^{T_i} y_{it}$, $\big(\sum_{t=1}^{T_i} X_{it,1}, \ldots, \sum_{t=1}^{T_i} X_{it,k}\big)^T$ and $\sum_{t=1}^{T_i} \epsilon_{it}$ correspondingly. Then we use $\epsilon_{it} - \bar{\epsilon} = (y_{it} - \bar{y}_i) - (X_{it} - \bar{X}_i)^T\beta$.

Taking maximum likelihood estimation, we can get least square estimator

for σ^2 is,

$$\hat{\sigma}^2 = \frac{1}{\sum_{i=1}^{n}(T_i - 1)} \sum_{i=1}^{n} \sum_{t=1}^{T_i} \left[(y_{it} - \bar{y_i}) - (X_{it} - \bar{X_i})^T \hat{\beta} \right]^2 \tag{4.33}$$

To estimate the variance of random effect, we analyze the model for all individuals, then we have $\bar{y_i} = \bar{X_i}'\beta + \alpha_i + \bar{\epsilon_i}$ and $\bar{y} = \bar{X}'\beta + \bar{\alpha} + \bar{\epsilon}$, where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} \bar{y_i}$, $\bar{\alpha} = \frac{1}{n}\sum_{i=1}^{n} \bar{\alpha_i}$, $\bar{\epsilon} = \frac{1}{n}\sum_{i=1}^{n} \bar{\epsilon_i}$ and $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} \bar{X_{i,1}}, \ldots, \frac{1}{n}\sum_{i=1}^{n} \bar{X_{i,k}})'$. Then we use $\bar{y_i} - \bar{y} = (\bar{X_i} - \bar{X})\beta + \alpha_i - \bar{\alpha} + \bar{\epsilon_i} - \bar{\epsilon}$, and take variance of both sides:

$$Var(\bar{y_i} - \bar{y}) = Var(\alpha_i - \bar{\alpha}) + Var(\bar{\epsilon_i} - \bar{\epsilon}) \tag{4.34}$$

and

$$\begin{aligned}
Var(\alpha_i - \bar{\alpha}) &= Var(\alpha_i - \frac{1}{n}\alpha_i - \frac{1}{n}\sum_{j \neq i}^{n} \alpha_j) = \big(1 - \frac{1}{n}\big)^2 \sigma_\alpha^2 + \frac{1}{n^2}(n-1)\sigma_\alpha^2 \\
&= \frac{n-1}{n}\sigma_\alpha^2 \\
Var(\bar{\epsilon_i} - \bar{\epsilon}) &= Var(\bar{\epsilon_i} - \frac{1}{n}\bar{\epsilon_i} - \frac{1}{n}\sum_{j \neq i}^{n} \bar{\epsilon_j}) = \big(1 - \frac{1}{n}\big)^2 Var(\bar{\epsilon_i}) + \frac{1}{n^2}\sum_{j \neq i}^{n} Var(\bar{\epsilon_j}) \\
&= \big(1 - \frac{1}{n}\big)^2 \frac{1}{T_i}\sigma^2 + \frac{1}{n^2}\sigma^2 \sum_{j \neq i}^{n} \frac{1}{T_j} \\
&= \big(1 - \frac{2}{n}\big)\frac{1}{T_i}\sigma^2 + \frac{\sigma^2}{n^2}\sum_{i=1}^{n} \frac{1}{T_i}
\end{aligned} \tag{4.35}$$

where $Var(\bar{\epsilon_i}) = Var(\frac{1}{T_i}\sum_{t=1}^{T_i} \epsilon_{it}) = \frac{1}{T_i^2} T_i \sigma^2 = \frac{1}{T_i}\sigma^2$

Merging equation (4.35) into (4.34): $\frac{n-1}{n}\sigma_\alpha^2 = Var(\bar{y_i} - \bar{y}) - Var(\bar{\epsilon_i} - \bar{\epsilon})$.

Then we can get the estimator of σ_α^2:

$$\begin{aligned}\hat{\sigma}_\alpha^2 &= \frac{n}{n-1}\frac{1}{n}\sum_{i=1}^{n}\left(\bar{y}_i-\bar{y}\right)^2-\frac{n}{n-1}\frac{1}{n}\left((1-\frac{2}{n})\sum_{j=1}^{n}\frac{1}{T_j}+n\frac{1}{n^2}\sum_{i=1}^{n}\frac{1}{T_i}\right)\hat{\sigma}^2 \\ &= \frac{1}{n-1}\sum_{i=1}^{n}\left(\bar{y}_i-\bar{y}\right)^2-\frac{1}{n}\sum_{i=1}^{n}\frac{1}{T_i}\hat{\sigma}^2 \end{aligned} \tag{4.36}$$

Finally, with estimation of β, σ^2 and σ_α^2, we can estimate the subject-specific random effect α_i. Consider the best linear unbiased predictor (Henderson, 1975), which show that, the mean square error $E||\hat{\alpha}-\alpha||^2$ is minimized by all predictors $\hat{\alpha}=\hat{\alpha}(y)$ of α based on y, at here α is a vector with the element of all α_i. In particular,

$$\hat{\alpha}_{BLUP} = \underset{\hat{\alpha}}{argmin}\ E||\hat{\alpha}-\alpha||^2$$

See Lai and Xing (2008),

$$\hat{\alpha} = \Sigma_{21}\Sigma_{11}^{-1}(y-X\beta) \tag{4.37}$$

where Σ_{21} and Σ_{11} is defined by

$$\begin{aligned}\Sigma_{21} = Cov(y,\alpha)' &= \begin{bmatrix} \underbrace{\sigma_\alpha^2,\ldots,\sigma_\alpha^2}_{T_1} & 0 & \cdots \\ & \ddots & \\ 0 & \cdots & \underbrace{\sigma_\alpha^2,\ldots,\sigma_\alpha^2}_{T_n} \end{bmatrix} \\ \Sigma_{11} = Cov(y) &= diag(V_1,\ldots,V_n) \end{aligned} \tag{4.38}$$

where $V_i=\sigma^2 I_{T_i}+\sigma_\alpha^2 J_{T_i}$, and y here is the vector of explained variables for all individuals in each period, α is the vector of random effect estimators.

With $\hat{\beta}_{GLS}$, we get the best linear unbiased predictor,

$$\hat{\alpha}_{BLUP} = \Sigma_{21}\Sigma_{11}^{-1}(y - X\hat{\beta}_{GLS}) \tag{4.39}$$

Then, the fitted values are,

$$\hat{y}_{it} = X'_{it}\hat{\beta} + \hat{\alpha}_i \tag{4.40}$$

4.4 Feedback Effect

In order to investigate the feedback effect of sovereign rating, we introduce a group of criteria to measure the movement in rating history and define them as indicator variables. To be more specific, they are shown as follows:

$$Z_{it,1} = \begin{cases} 1 & y_{i,t-1} > y_{i,t-2} \\ 0 & y_{i,t-1} = y_{i,t-2} \\ 0 & y_{i,t-1} < y_{i,t-2} \end{cases}$$

where Z_{it} is the explanatory variable which present history movement, for individual i at period t, $y_{i,t}$ here is the numerical rating level for individual i at period t and $Z_{it,1}$ here means positive rating history movement.

Similarly,

$$Z_{it,2} = \begin{cases} 0 & y_{i,t-1} > y_{i,t-2} \\ 0 & y_{i,t-1} = y_{i,t-2} \\ 1 & y_{i,t-1} < y_{i,t-2} \end{cases} \qquad Z_{it,3} = \begin{cases} 0 & y_{i,t-1} > y_{i,t-2} \\ 1 & y_{i,t-1} = y_{i,t-2} \\ 0 & y_{i,t-1} < y_{i,t-2} \end{cases}$$

where $Z_{it,2}$ here means stable rating history and $Z_{it,3}$ here means negative rating history movement. Notice that $Z_{it,1} + Z_{it,2} + Z_{it,3} = 1$ for any individual at a certain period.

Moreover, we introduce a lag-feedback effect variable for advanced analysis, it can be denoted similarly as feedback effect variable, which is shown as follows:

$$W_{it,1} = \begin{cases} 1 & y_{i,t-2} > y_{i,t-3} \\ 0 & y_{i,t-2} = y_{i,t-3} \\ 0 & y_{i,t-2} < y_{i,t-3} \end{cases}$$

where W_{it} is the explanatory variable which lagged history movement, for individual i at period t. Similarly,

$$W_{it,2} = \begin{cases} 0 & y_{i,t-2} > y_{i,t-3} \\ 0 & y_{i,t-2} = y_{i,t-3} \\ 1 & y_{i,t-2} < y_{i,t-3} \end{cases} \qquad W_{it,3} = \begin{cases} 0 & y_{i,t-2} > y_{i,t-3} \\ 1 & y_{i,t-2} = y_{i,t-3} \\ 0 & y_{i,t-2} < y_{i,t-3} \end{cases}$$

As we see, $Z_{it,1} + Z_{it,2} + Z_{it,3} = 1$, $W_{it,1} + W_{it,2} + W_{it,3} = 1$, any one of the feedback variable can be write as linear combination of other two. So, when carrying out the empirical analysis, we use two of the three variables.

Then general model to estimate can be developed from the model 4.3:

$$y_i = \alpha + X_i'\beta + \gamma_1 Z_{i,1} + \gamma_2 Z_{i,2} + \delta_1 W_{i,1} + \delta_2 W_{i,2} + \epsilon_i \tag{4.41}$$

It is convenience to write a as $ax0_{it}$, where $x0 = 1$, and the indicator variables, Z_i, W_i can be included in the explanatory variable set X_i. Then the regression model 4.3 can be written in matrix form as

$$\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{4.42}$$

where

$$\boldsymbol{Y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} \alpha \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix}, \quad \boldsymbol{\epsilon} = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_n \end{pmatrix}, \quad \boldsymbol{X} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix}$$

with $y_i^{T_i \times 1} = (y_{i1}, \ldots, y_{iT_i})'$, $X_i^{T_i \times (k+5)} = (X_{i1}^{(k+5)\times 1}, \ldots, X_{iT_i}^{(k+5)\times 1})'$, $\epsilon_i^{T_i \times 1} = (\epsilon_{i1}, \ldots, \epsilon_{iT_i})'$

The vector of least squares estimates of the β is given by,

$$\hat{\beta_{OLS}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{Y} \tag{4.43}$$

Then we can estimate α by

$$\hat{\alpha}_{OLS} = \bar{\boldsymbol{Y}} - \bar{\boldsymbol{X}}'\hat{\beta}_{OLS} \tag{4.44}$$

Chapter 5 Empirical analysis

We build a rating database with sovereign foreign currency rating attributed by Standard & Poor's and Moody's. For the rating notations we covered the period from 1980 to 2016. The quarterly rating level we used is the rating that was attributed at last day of corresponding quarter. Taking the data availability of the explanatory covariates into consideration, our empirical analysis only cover the period from 1993 to 2016. Fiscal balance and external balance are in percentage of GDP, debt service ratio is in percentage of export. Overall we have an unbalance panel with 67 countries for Standard & Poor's, 66 countries for Moody's and 60 countries for Fitch's data set.

Two sets of variables are used in the analysis, economic covariates and indicator covariates. Firstly, macroeconomic and finance covariates include

- GDP per capita: we take the natural logarithm value of GDP per capita PPP current international dollar.
- Real GDP: we take the natural logarithm value of real GDP.
- GDP growth rate: we use the annual percentage growth rate of GDP per capita.
- Inflation: we use the annual change of consumer price index.
- Fiscal balance: we take the percentage of government budget balance in GDP.
- External balance: we take the percentage of current account balance in GDP.
- Debt service ratio: we take the percentage of external debt in export.

Then, the indicator covariates include

- Default history: The default event record, 1 means the country has defaulted history on its issued bond, 0 means default event never happen.
- Economic development I: The industrialized classification, 1 means the country is industrialized one and 0 for non-industrialized countries .
- Economic development II: The least developed countries classification, 1 means the country is least developed and 0 for non least developed countries.
- Geopolitics I: Group of seven(G7) classification, 1 means the country is a member of G7, 0 means it's not in the membership.
- Geopolitics II: Group of twenty(G20) classification, 1 means the country is a member of G20, 0 means it's not in the membership.
- Geopolitics III: OECD classification, 1 means the country is a member of OECD, 0 means it's not in the membership.
- Sociology: The west country classification: 1 means the country rooted in and derived by European cultures, 0 means it doesn't.

As the first step, we set the macroeconomic and finance covariates as baseline data set to estimate the fixed and random effect parameters. Then we introduce the indicator variables into analysis term by term to check the influence on parameter estimation.

Like most previous literature, we transform sovereign ratings into data for regression analysis. To be more specific, we assign numerical values to the credit rating agencies' criteria as follows: $AAA/Aaa = 8$, $AA+, AA, AA-/Aa1, Aa2, Aa3 = 7$ and so on through $D = 1$. I.e, a higher rating level comes with a higher numerical value.

5.1 Linear Regression

Generally speaking, the table 5.1 shows that the variables we selected have significant influence on dependent variable. While, note that, we classify industrialization and 'Least Developed country' into 'Economic Development' indicator variable.

In regular linear regression, as conducted in this part, we treat the intercept part as a common factor. The subject-specific influences are absorbed into the estimation of global parameters, which makes the results can't reveal the potential individual-specific effect.

Table 5.1: Estimation of linear regression, with S&P, Moody and Fitch Data.

Begin of Table			
	S&P's Rating	Moody's Rating	Fitch's Rating
Intercept	-14.338 *** [1]	-18.401 ***	-13.905 ***
	(-44.727) [2]	(-48.013)	(-35.970)
Econ. & Fin. Covar.			
Per capita income	1.265 ***	1.371 ***	1.368 ***
	(50.500)	(48.271)	(39.259)
Real GDP	0.286 ***	0.395 ***	0.287 ***
	(23.922)	(29.130)	(18.596)
GDP growth rate	0.056 ***	0.058 ***	0.046 ***
	(10.291)	(9.393)	(6.937)
Inflation	-0.029 ***	-0.002 **	-0.025 ***
	(-15.748)	(-2.661)	(-11.877)
Fiscal Balance	0.038 ***	0.042 ***	0.032 ***
	(7.172)	(6.957)	(4.886)
External Balance	-0.004 *	-0.011 **	-0.006
	(-1.302)	(-3.005)	(-1.483)
Debt ratio	-0.006 ***	-0.004 ***	-0.006 ***
	(-4.119)	(-2.738)	(-3.414)
Indicator Covar.			
Default History	-1.397 ***	-1.366 ***	-1.569 ***
	(-11.926)	(-10.739)	(-10.619)
Industrialization	1.013 ***	0.981 ***	0.391 ***
	(19.639)	(16.644)	(5.156)
Least Developed Country	-1.529 ***	-1.640 ***	-2.477 ***
	(-8.886)	(-4.344)	(-9.514)
Group of Seven (G7)	0.809 ***	0.628 ***	0.904 ***
	(11.449)	(7.748)	(12.136)
Group of Twenty (G20)	-0.778 ***	-0.897 ***	-0.714 ***

[1] . Significant at the 10 percent level; * Significant at the 5 percent level
** Significant at the 1 percent level; *** Significant at the 0.1 percent level
[2] The t-statistics are in parentheses

Continuation of Table 5.1			
	S&P's Rating	Moody's Rating	Fitch's Rating
	(-14.493)	(-15.040)	(-10.157)
OECD	1.124 ***	1.101 ***	1.486 ***
	(23.652)	(21.427)	(25.588)
Sociology	1.542 ***	1.579 ***	1.906 ***
	(34.700)	(33.271)	(37.476)
Adjusted R-squared	0.735	0.687	0.730
Standard error	0.988	1.125	1.007
End of Table			

As an initial analysis, we can see the most of the parameter seems reasonable to anticipate the signs. In particular, a high per capita income appears to be closely related to the high ratings, so as the real GDP, GDP growth rate, fiscal balance and external balance. Inflation and default history make negative influence on rating assignment. To be more specific, as indicators of economic performance, GDP and GDP growth rate, provide the country tax and fiscal bases and health economy structure to repay the existing debt burden. And, the government budget balance determines a sovereign's flexibility, it reveals the sovereign's ability to repay the debt from their tax revenue.

By contrast, Inflation, an indicator to measure issuer's credibility of its monetary policy, reflect structure problems in the government's finances, and as a further step, a relative high inflation rate threaten the ability to repay the sovereign debt. External debt, in our study which is introduced by debt ratio, reveals that higher debt burden correspond to a lower assigned rating. Another negative indicator on rating assignment, the default history perceived as a crucial signal of credit risk.

As economic development indicator variable, the countries, which is classified as industrialization and non-least developed, have a higher rating than the ones with lower economic development. And, we can also see, the membership in some economic and finance cooperation organization helps the countries get better rating level.

It is interesting to point also to the signs for the explanatory variable, External Balance, is different from expectation and existing literature. A current balance surplus makes the issuers receive relatively low rating. Moreover, we

should notice that this explanatory variable loses its statistical significance under analysis with Fitch data.

The results for all ratings are shown in Table 5.1 and allow us to conclude that all the coefficients have the expected signs, which is discussed in 'determinants in sovereign ratings' section, except for the external balance. And, most of the coefficients are indeed statistically different from zero. What's more, the results assesses that whether S&P, Moody or Fitch are using the same explanatory variables when they indicative rating levels. This is tested by pooling the data for all ratings and by running a common linear regression.

5.2 Estimation of Parameter under Fixed Effect

As the empirical result with fixed effect model. We firstly discuss the estimation of common parameter, economic and financial indicators. Then we move on to the country-specific parameters.

5.2.1 Common Parameter

The table 5.2 shows that the estimation values of each parameter have same signs, with the comparison to linear regression model. In particular, real GDP, GDP growth and fiscal balance have negative relationship with the numerical rating level, which means that a higher real GDP, GDP growth rate and fiscal balance appear to be closely related to high rating. On the contrary, a higher inflation, external balance and debt service ratio relate to lower rating.

Table 5.2: Estimation of common coefficient under fixed effect with GLS, with S&P, Moody and Fitch Data

	S & P's Rating	Moody's Rating	Fitch's Rating
Econ. & Fin. Covariates			
Per capita income	0.0799	0.3219	0.3607
Real GDP	1.0198	0.597	0.5387
GDP growth rate	0.0209	0.0068	0.0103
Inflation	-0.0087	-0.0003	-0.0068
Fiscal Balance	0.038	0.0564	0.0531
External Balance	-0.0377	-0.0476	-0.0487
Debt ratio	-0.0047	-0.0061	-0.0046
Indicator Covariates			
Default History	-0.6314	-0.1616	0.188
Industrialization	-1.0069	-13.3669	0.0135
Least Developed Country	1.3134	-13.3672	-0.8158
Group of Seven (G7)	-1.0069	-0.9736	0.5973
Group of Twenty (G20)	-0.2916	-0.2358	0.5973
OECD	-1.0069	0.5298	0.0368
Sociology	-1.0068	0.3739	0.0368

Focus on the table 5.2, the estimated parameter for the indicator variables show difference among the rating agencies. Especially in the indicator covariates part, the sign and weight of these geopolitics and development variables are different.

5.2.2 Estimation of Fixed Effect

In this part, we show the empirical result analyzed with S&P, Moody and Fitch data case by case. And for each case, we illustrate with the help of data map, which show the result in a worldwide map, clustered bar and histogram bar.

S&P data

As the first part, we introduce the result with S&P data. In Table 5.3, we can see the estimation of country-specific fixed effect. It show that each country get a different rating scale even the rating assignment is under a common parameter set.

Table 5.3: Estimation of fixed effect coefficient with GLS, with S&P Data

Country	Fix. Eff.	Country	Fix. Eff.	Country	Fix. Eff.
Argentina	-2.123	Hungary	0.863	Peru	0.114
Australia	1.558	Iceland	3.477	Philippines	-0.267
Austria	2.656	India	-1.917	Poland	0.144
Belgium	1.690	Indonesia	-2.036	Portugal	1.047
Bolivia	1.283	Ireland	2.405	Romania	-0.241
Brazil	-2.622	Italy	-0.835	Russia	-2.024
Bulgaria	1.153	Jamaica	0.677	Rwanda	1.281
Cameroon	0.268	Japan	-0.913	Saudi Arabia	1.471
Canada	1.342	Kazakhstan	0.519	Singapore	4.119
Chile	1.459	Kenya	-0.186	Slovenia	3.033
China	-1.849	Latvia	2.481	South Africa	-0.325
Colombia	-0.475	Lebanon	-0.422	South Korea	-0.368
Czech Rep.	1.738	Lithuania	2.368	Spain	0.298
Denmark	3.141	Luxembourg	5.647	Sri Lanka	-0.320
Dom. Rep.	0.149	Mexico	-1.463	Thailand	0.381
Ecuador	-0.939	Mongolia	1.955	Turkey	-1.982
Egypt	-0.682	Morocco	0.527	Uganda	0.282
Estonia	3.600	Netherlands	2.232	Ukraine	-1.110
Finland	3.168	New Zea.	2.604	United Kingdom	1.099
France	0.657	Nigeria	-1.618	United States	-1.007
Germany	0.826	Oman	2.814	Uruguay	1.313
Ghana	-0.280	Pakistan	-1.380		
Greece	-0.453	Paraguay	1.066		

In Figure 5.1, we illustrate the estimation of fixed effect with S&P data, and with the map, we can observe the results intuitively.

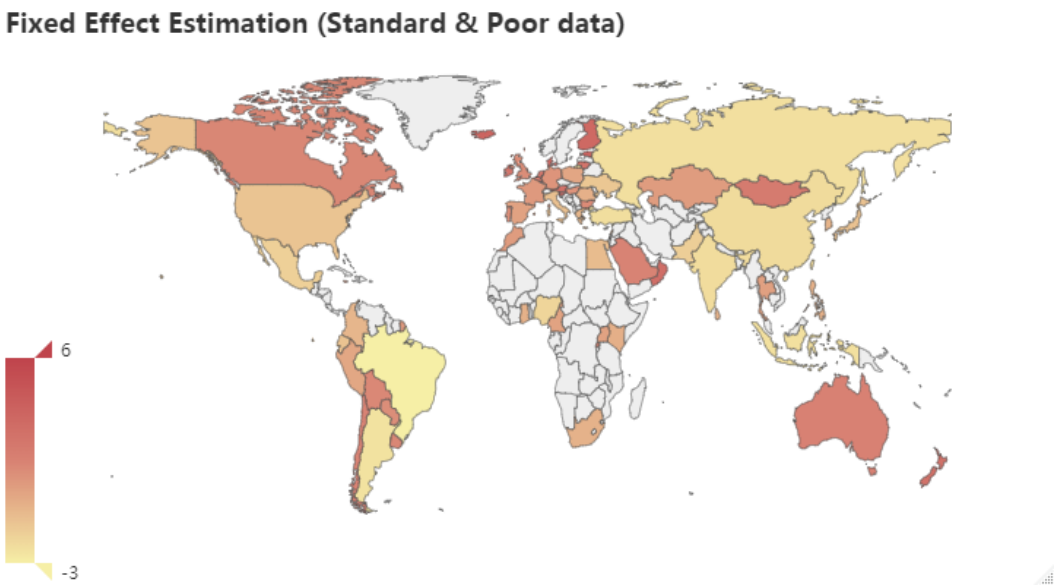


Figure 5.1: Estimation of Fixed Effect with S&P data

We can see that, countries in Northern Europe, Oceania and Middle East come along with high-level of subject-specific parameter. These countries share several similar characters, as high performance in economics and finance, stable political structure or geopolitics patterns. For example, several European countries get significant improvement, as Denmark, Finland, Iceland and Luxembourg. Several high developed country also get the benefit, like Singapore. On the contrary, most of countries in South America and Asia get relatively lower level rating assignment.

With the help of histogram and clustered figure 5.2, we get information about the distribution of data.

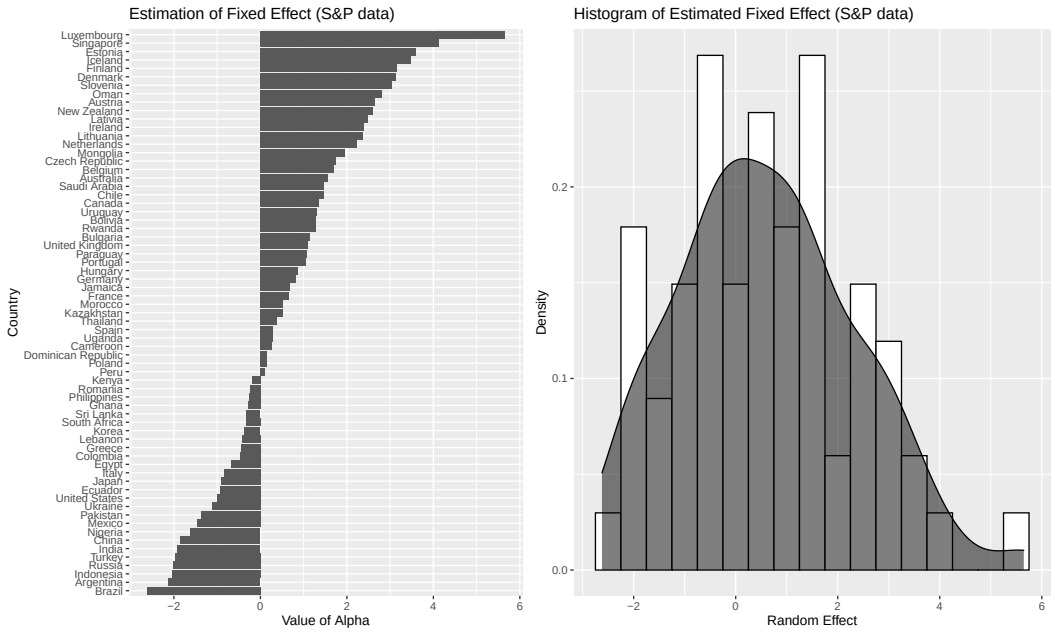


Figure 5.2: Histogram of Fixed Effect with S&P data

Moody data

In this part, we introduce the result with Moody data. In Table 5.4, we can see the estimation of country-specific fixed effect. It show that each country get a different rating scale even the rating assignment is under a common parameter set.

Table 5.4: Estimation of fixed effect coefficient with GLS, with Moody Data

Country	Fix. Eff.	Country	Fix. Eff.	Country	Fix. Eff.
Argentina	-1.951	Greece	-0.924	Pakistan	-1.144
Australia	0.849	Hungary	0.105	Paraguay	-0.452
Austria	1.439	Iceland	1.164	Peru	-0.304
Belgium	0.736	India	-0.641	Philippines	-0.545
Bolivia	0.082	Indonesia	-1.156	Poland	0.036
Brazil	-1.870	Ireland	0.802	Portugal	0.198
Bulgaria	-0.215	Italy	-0.320	Romania	-0.640
Cameroon	-0.403	Jamaica	-0.972	Russia	-1.200
Canada	0.826	Japan	0.142	Saudi Arabia	0.030
Chile	0.271	Kazakhstan	-0.256	Singapore	2.209
China	-0.088	Kenya	-0.748	Slovenia	0.981
Colombia	-0.594	Latvia	0.855	South Africa	-0.193
Czech Rep.	0.298	Lebanon	-1.392	South Korea	-0.270
Denmark	1.631	Lithuania	0.498	Spain	0.252
Dom. Rep.	-0.809	Luxembourg	2.781	Sri Lanka	-1.037
Ecuador	-1.558	Mexico	-0.914	Thailand	-0.080
Egypt	-0.809	Mongolia	-0.429	Turkey	-1.607
Estonia	1.071	Morocco	-0.339	Uganda	-0.479
Finland	1.618	Netherlands	1.424	Ukraine	-1.306
France	0.638	New Zea.	1.661	United Kingdom	0.784
Germany	0.757	Nigeria	-0.984	United States	-0.048
Ghana	-1.071	Oman	0.953	Uruguay	-0.049

In Figure 5.3, Intuitively, we illustrate the estimation of fixed effect analyzed by Moody data with help the worldwide map.

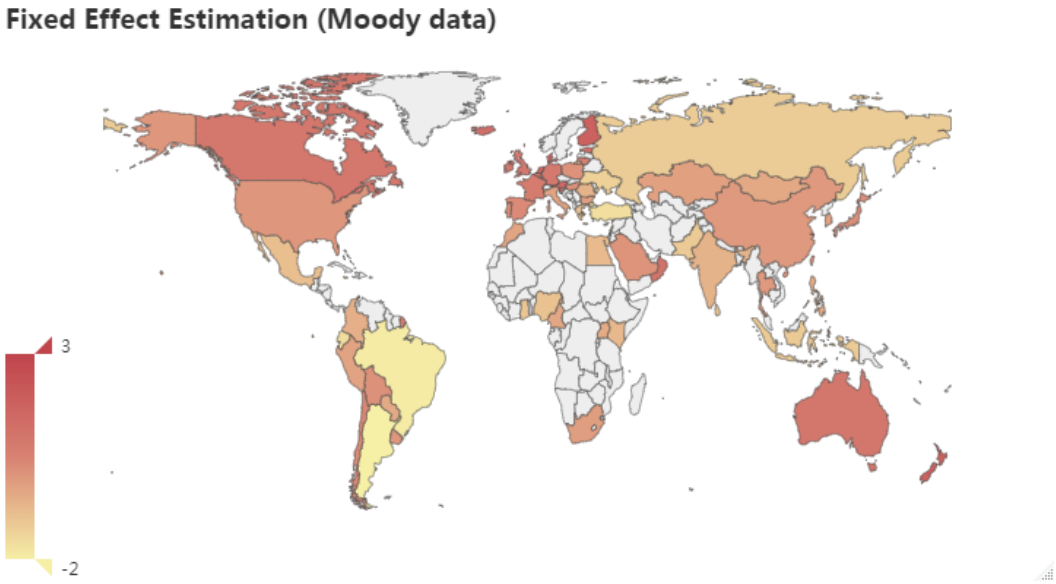


Figure 5.3: Estimation of Fixed Effect with Moody data

Similarly with S&P result, with the analysis under Moody's data, countries in Northern Europe, Oceania and Middle East receive high-level of subject-specific parameter. In the meanwhile, we can see some countries still suffer high-level negative influence from the subject-specific parameter, they are Argentina, Brazil, China, India, Indonesia, Mexico, Nigeria, Russia, Turkey, Uruguay, most of which are developing countries.

With the help of histogram and clustered figure 5.4, we get information about the distribution of data.

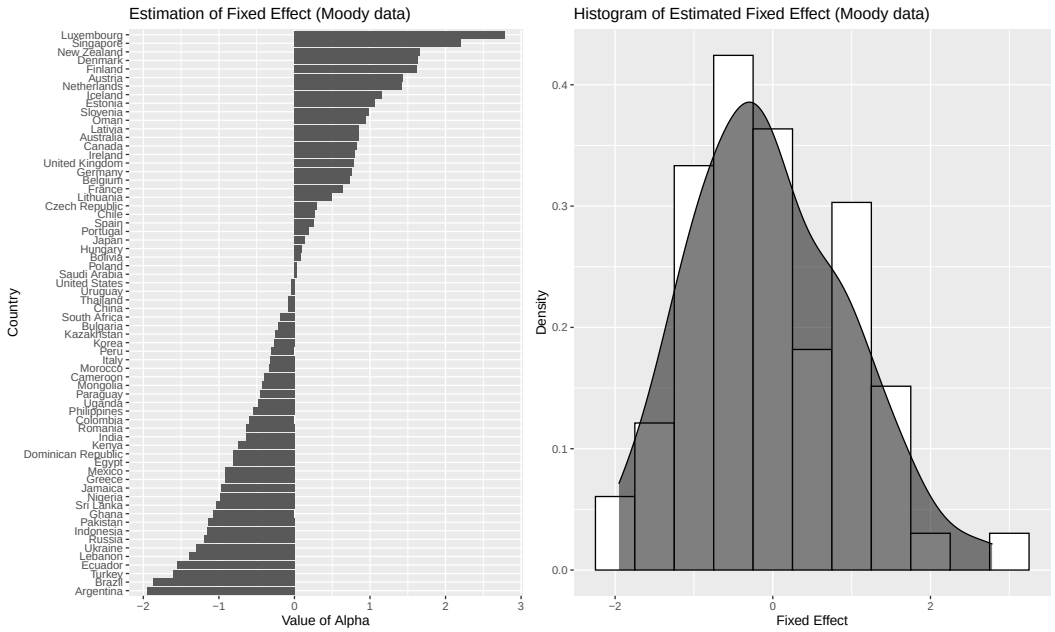


Figure 5.4: Histogram of Fixed Effect with Moody data

Fitch data

In this part, we introduce the result with Moody data. In Table 5.5, we can see the estimation of country-specific fixed effect. It show that each country get a different rating scale even the rating assignment is under a common parameter set.

Table 5.5: Estimation of fixed effect coefficient with GLS, with Fitch Data

Country	Fix. Eff.	Country	Fix. Eff.	Country	Fix. Eff.
Argentina	-2.459	Greece	-0.790	Nigeria	-0.639
Australia	0.549	Hungary	-0.033	Pakistan	-1.331
Austria	1.711	Iceland	0.726	Paraguay	0.112
Belgium	0.895	India	-0.551	Peru	-0.240
Bolivia	0.084	Indonesia	-1.151	Philippines	-0.267
Brazil	-1.558	Ireland	1.115	Poland	-0.055
Bulgaria	0.012	Italy	-0.178	Portugal	0.276
Cameroon	-0.387	Jamaica	-1.028	Romania	-0.490
Canada	0.615	Japan	-0.145	Russia	-1.161
China	-0.294	Kazakhstan	-0.192	Rwanda	0.444
Czech Rep.	0.520	Kenya	-0.593	Slovenia	1.230
Denmark	1.853	Latvia	0.714	South Korea	-0.151
Dom. Rep.	-1.218	Lebanon	-1.511	Spain	0.356
Ecuador	-1.411	Lithuania	0.502	Sri Lanka	-0.399
Egypt	-0.597	Luxembourg	2.867	Turkey	-1.337
Estonia	1.139	Mexico	-0.843	Uganda	-0.092
Finland	1.881	Mongolia	-0.486	Ukraine	-1.153
France	0.901	Morocco	0.236	United Kingdom	0.954
Germany	1.008	Netherlands	1.664	United States	0.134
Ghana	-0.858	New Zea.	1.149	Uruguay	-0.047

In Figure 5.5, we illustrate the estimation of fixed effect with Fitch data, and with the map, we can observe the results intuitively.

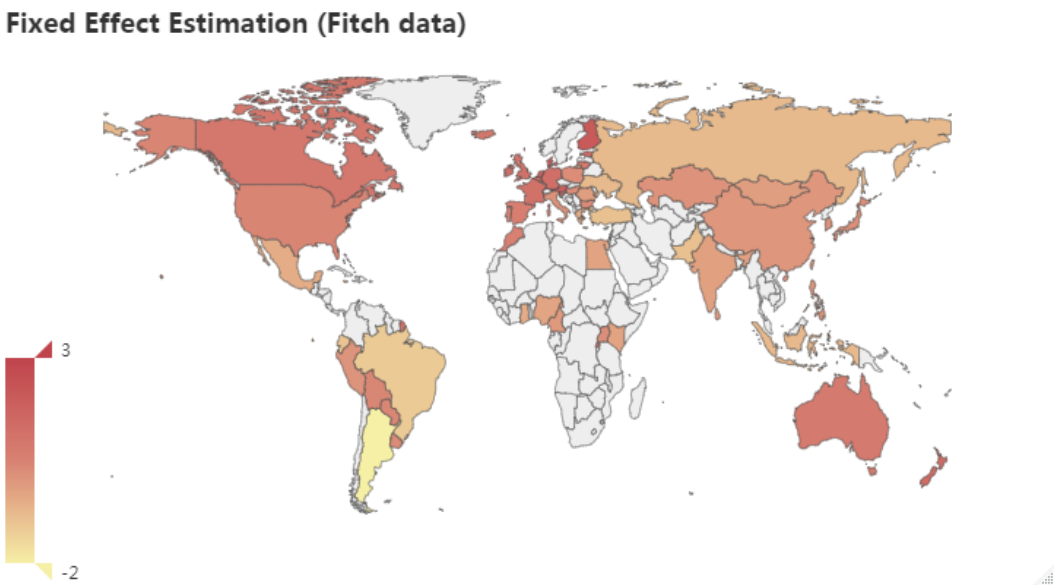


Figure 5.5: Estimation of Fixed Effect with Fitch data

Slightly different from the result above, United States receive a relatively-high rating assignment compared with the other two rating agencies. And countries in Northern Europe, Oceania and Middle East receive high-level of subject-specific parameter.

With the help of histogram and clustered figure 5.6, we get information about the distribution of data.

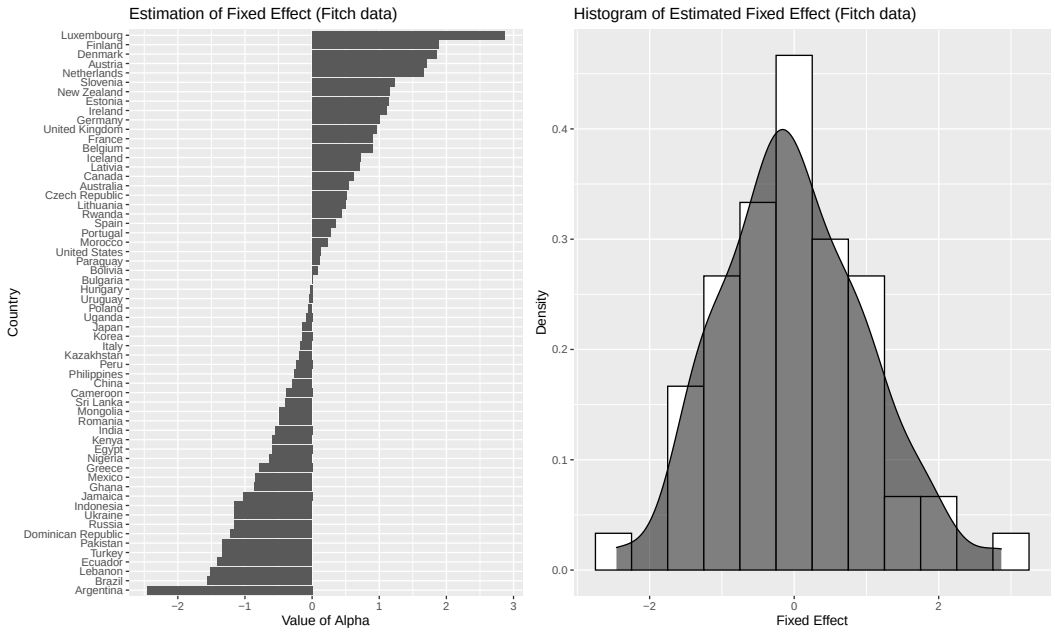


Figure 5.6: Histogram of Fixed Effect with Fitch data

Comparing the results of analysis with three agencies' data, there are some meaningful findings. The countries experienced financial crisis in currency or bond, all got low estimated fixed effect value, like Argentina, Brazil, Greece, Italy, Turkey and Russia. Even though some of them locates in Europe. And there is bias problem in result. For example, China receive a relative-high estimated value from Moody, and the estimated fixed effect value for United States is much lower in the result with S&P data.

5.3 Estimation of Parameter under Random Effect

As the empirical result with random effect model. We firstly discuss the estimation of common parameter, economic and financial indicators. Then we move on to the country-specific parameters.

5.3.1 Common Parameter

The table 5.6 shows that the estimation values of each parameter have same signs, with the comparison to linear regression model. In particular, GDP growth and fiscal balance have positive relationship with the numerical rating level, which means that a higher real GDP, GDP growth rate and fiscal balance appear to be closely related to high rating. On the contrary, a higher inflation, external balance and debt service ratio relate to lower rating. It is noteworthy that, the signs of explanatory variable, Real GDP, in Moody and Fitch's analysis, are different from linear regression model and fixed effect model.

Table 5.6: Estimation of common coefficient under random effect with GLS, with S&P, Moody and Fitch Data

	S & P's Rating	Moody's Rating	Fitch's Rating
Econ. & Fin. Covariates			
Per capita income	0.0096	0.7695	0.6308
Real GDP	0.9126	-0.0535	-0.0005
GDP growth rate	0.0213	0.0084	0.0113
Inflation	-0.0084	-0.0001	-0.0075
Fiscal Balance	0.038	0.0563	0.0531
External Balance	-0.0382	-0.0498	-0.0515
Debt ratio	-0.0048	-0.0069	-0.0047
Indicator Covariates			
Default History	-0.6299	-0.1676	-0.0397
Industrialization	1.2431	0.968	0.017
Least Developed Country	0.4489	-9.6374	-0.7995
Group of Seven (G7)	0.1043	1.5426	0.7824
Group of Twenty (G20)	-0.2911	-0.1768	0.7824
OECD	1.2428	0.5015	0.0725
Sociology	1.7825	0.3403	0.0852

Focus on the table 5.6, the estimated parameter for the indicator variables show difference among the rating agencies. Under the random effect analysis, the sign and weight of the economic development and geopolitical variable are almost the same, which is different from the fixed effect analysis, except for G-20 classification.

5.3.2 Estimation of Random Effect

In this part, we show the empirical result under random effect analyzed with S&P, Moody and Fitch data case by case. And for each case, we illustrate with the help of data map, which show the result in a worldwide map, clustered bar and histogram bar.

S&P data

As the first part, we introduce the result with S&P data. In Table 5.7, we can see the estimation of country-specific random effect. It shows that each country get a different rating scale even the rating assignment is under a common parameter set.

Table 5.7: Estimation of random effect coefficient with GLS, with S&P Data

Country	Ran. Eff.	Country	Ran. Eff.	Country	Ran. Eff.
Argentina	-2.354	Hungary	0.496	Peru	-0.177
Australia	1.362	Iceland	2.802	Philippines	-0.476
Austria	2.345	India	-1.879	Poland	-0.084
Belgium	1.406	Indonesia	-2.138	Portugal	0.719
Bolivia	0.834	Ireland	2.025	Romania	-0.550
Brazil	-2.654	Italy	-0.946	Russia	-2.108
Bulgaria	0.720	Jamaica	0.141	Rwanda	0.824
Cameroon	-0.097	Japan	-0.919	Saudi Arabia	1.192
Canada	1.186	Kazakhstan	0.163	Singapore	3.710
Chile	1.164	Kenya	-0.496	Slovenia	2.591
China	-1.739	Latvia	1.953	South Africa	-0.531
Colombia	-0.702	Lebanon	-0.904	South Korea	-0.542
Czech Rep.	1.394	Lithuania	1.877	Spain	0.147
Denmark	2.813	Luxembourg	5.064	Sri Lanka	-0.702
Dom. Rep.	-0.263	Mexico	-1.573	Thailand	0.158
Ecuador	-1.305	Mongolia	1.358	Turkey	-2.147
Egypt	-0.933	Morocco	0.226	Uganda	-0.057
Estonia	3.036	Netherlands	2.000	Ukraine	-1.384
Finland	2.817	New Zea.	2.213	UK	1.004
France	0.565	Nigeria	-1.747	US	-0.942
Germany	0.758	Oman	2.296	Uruguay	0.839
Ghana	-0.633	Pakistan	-1.577		
Greece	-0.763	Paraguay	0.590		

Variance of estimated country-specific parameter in random effect model is 1.782.

In Figure 5.7, we illustrate the estimation of random effect with S&P data, and with the map, we can observe the results intuitively.

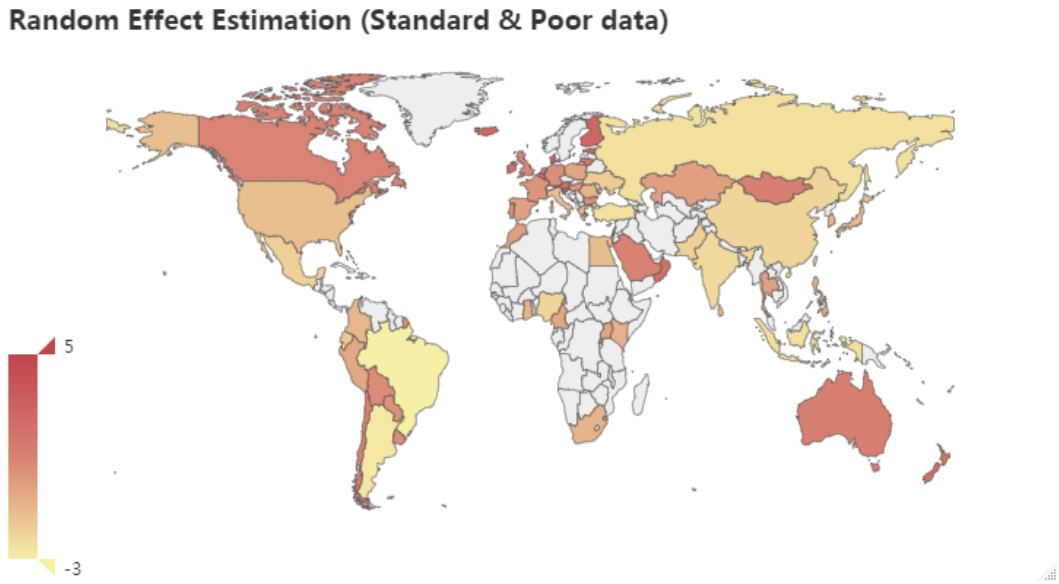


Figure 5.7: Estimation of Random Effect with S&P data

We can see that, countries in Northern Europe, Oceania and Middle East come along with high-level of subject-specific parameter. These countries share several similar characters, as high performance in economics and finance, stable political structure or geopolitics patterns. For example, several European countries get significant improvement, as Denmark, Finland, Iceland and Luxembourg. Several high developed country also get the benefit, like Singapore. On the contrary, most of countries in South American and Asian get relatively lower level rating assignment.

With the help of histogram and clustered figure 5.8, we get information about the distribution of data.

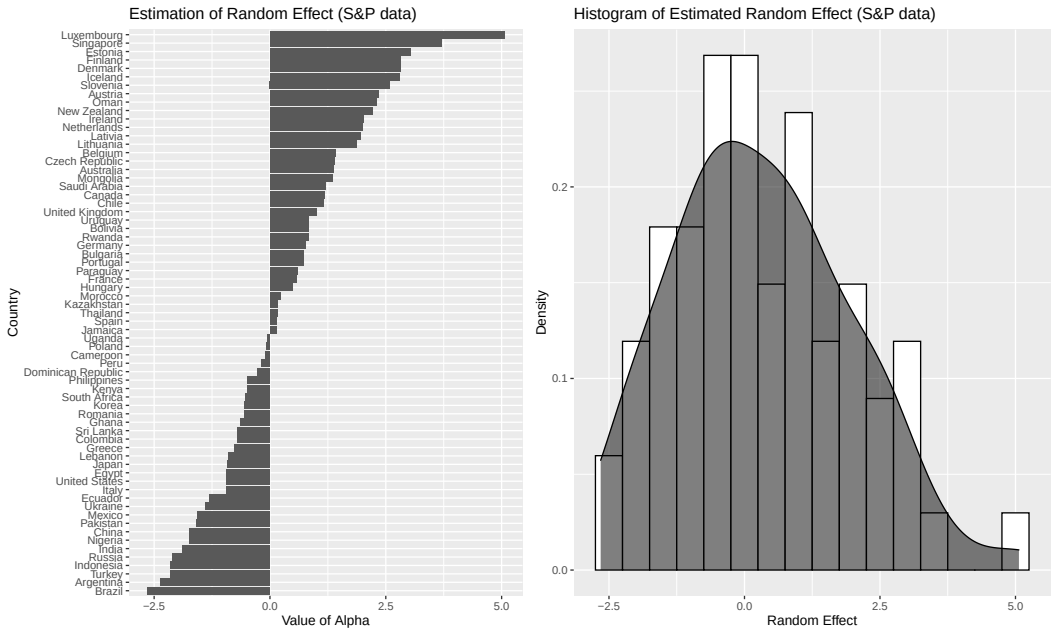


Figure 5.8: Histogram of Random Effect with S&P data

Moody data

In this part, we introduce the result with Moody data. In Table 5.8, we can see the estimation of country-specific fixed effect. It show that each country get a different rating scale even the rating assignment is under a common parameter set.

Table 5.8: Estimation of random effect coefficient with GLS, with Moody Data

Country	Ran. Eff.	Country	Ran. Eff.	Country	Ran. Eff.
Argentina	-3.024	Greece	-2.002	Pakistan	-1.838
Australia	1.192	Hungary	-0.946	Paraguay	-2.492
Austria	1.317	Iceland	-1.247	Peru	-1.143
Belgium	0.494	India	0.300	Philippines	-1.045
Bolivia	-1.596	Indonesia	-1.206	Poland	-0.208
Brazil	-1.725	Ireland	0.028	Portugal	-0.558
Bulgaria	-1.827	Italy	0.041	Romania	-1.659
Cameroon	-1.779	Jamaica	-3.540	Russia	-1.084
Canada	1.399	Japan	1.333	Saudi Arabia	-0.515
Chile	-0.319	Kazakhstan	-1.405	Singapore	1.833
China	1.521	Kenya	-1.936	Slovenia	-0.148
Colombia	-1.145	Lativia	-0.898	South Africa	-0.457
Czech Rep.	-0.519	Lebanon	-3.771	South Korea	-0.284
Denmark	1.480	Lithuania	-1.167	Spain	0.607
Dom. Rep.	-2.562	Luxembourg	1.656	Sri Lanka	-2.678
Ecuador	-3.337	Mexico	-0.854	Thailand	-0.410
Egypt	-1.592	Mongolia	-3.143	Turkey	-2.122
Estonia	-0.800	Morocco	-1.293	Uganda	-1.762
Finland	1.317	Netherlands	1.777	Ukraine	-2.441
France	1.520	New Zea.	1.120	United Kingdom	1.682
Germany	1.826	Nigeria	-1.116	United States	1.525
Ghana	-2.595	Oman	-0.625	Uruguay	-1.843

Variance of estimated country-specific parameter in random effect model is 1.822.

In Figure 5.9, intuitively, we illustrate the estimation of random effect

analyzed by Moody data through the worldwide map.

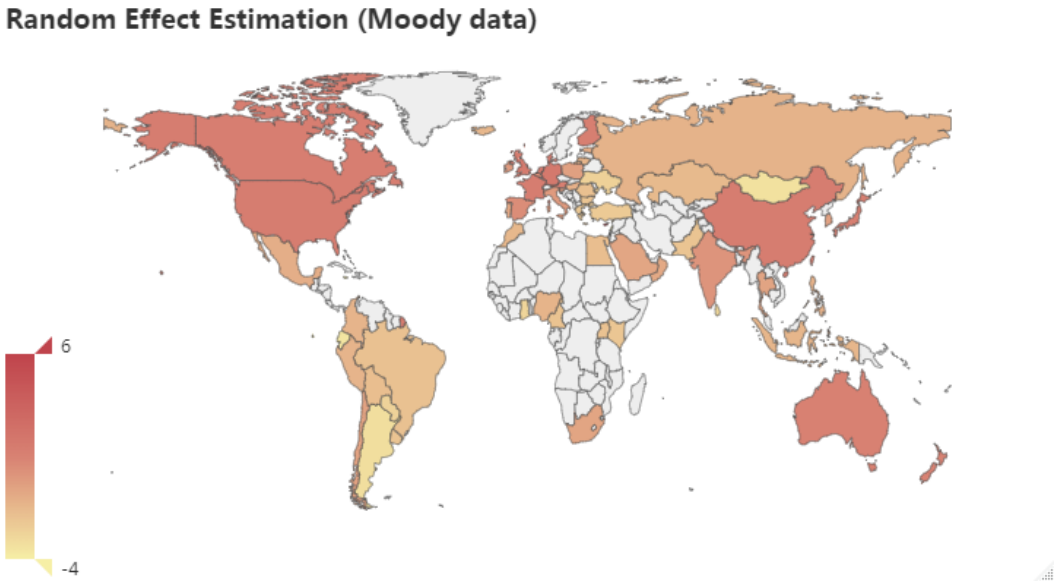


Figure 5.9: Estimation of Random Effect with Moody data

Similarly as S&P result, with the analysis under Moody's data, countries in Northern Europe, Oceania and Middle East receive high-level of subject-specific parameter. Slightly different from the result of S&P result, United States and China receive a relatively-high rating assignment. On the contrary, we can see some countries still suffer negative influence from the subject-specific parameter. These countries locate in Asia, Eastern Europe, South American and Africa , most of which are developing countries.

With the help of histogram and clustered figure 5.10, we get information about the distribution of data.

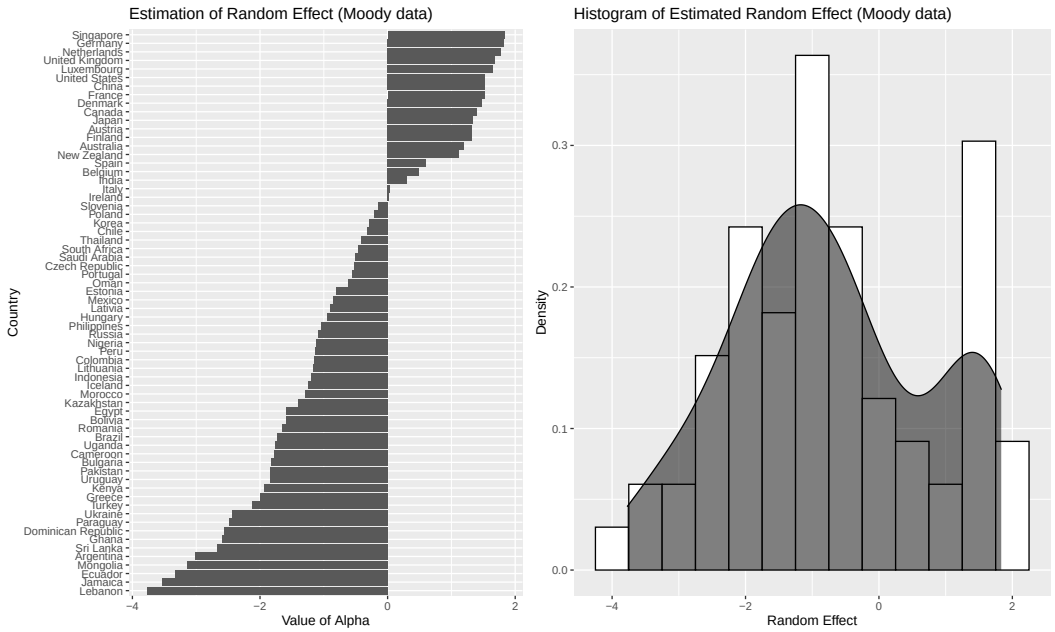


Figure 5.10: Histogram of Random Effect with Moody data

Fitch data

In this part, we introduce the result with Moody data. In Table 5.9, we can see the estimation of country-specific fixed effect. It show that each country get a different rating scale even the rating assignment is under a common parameter set.

Table 5.9: Estimation of random effect coefficient with GLS, with Fitch Data

Country	Ran. Eff.	Country	Ran. Eff.	Country	Ran. Eff.
Argentina	-3.645	Greece	-1.796	Nigeria	-0.971
Australia	0.571	Hungary	-1.148	Pakistan	-2.345
Austria	1.496	Iceland	-1.588	Paraguay	-1.640
Belgium	0.576	India	-0.087	Peru	-1.201
Bolivia	-1.631	Indonesia	-1.440	Philippines	-0.899
Brazil	-1.530	Ireland	0.403	Poland	-0.508
Bulgaria	-1.530	Italy	0.031	Portugal	-0.522
Cameroon	-1.975	Jamaica	-3.435	Romania	-1.516
Canada	0.003	Japan	0.607	Russia	-1.231
China	-0.765	Kazakhstan	-1.346	Rwanda	-1.696
Czech Rep.	-0.291	Kenya	-1.941	Slovenia	0.142
Denmark	1.601	Latvia	-1.054	South Korea	-0.292
Dom. Rep.	-3.008	Lebanon	-3.720	Spain	0.505
Ecuador	-3.056	Lithuania	-1.133	Sri Lanka	-1.842
Egypt	-1.432	Luxembourg	1.749	Turkey	-1.851
Estonia	-0.654	Mexico	-0.983	Uganda	-1.783
Finland	1.504	Mongolia	-3.025	Ukraine	-2.320
France	1.533	Morocco	-0.681	United Kingdom	1.545
Germany	1.802	Netherlands	1.851	United States	1.368
Ghana	-2.456	New Zea.	0.320	Uruguay	-1.773

Variance of estimated country-specific parameter in random effect model is 1.772.

In Figure 5.11, we illustrate the estimation of random effect with Fitch data, and with the map, we can observe the results intuitively.

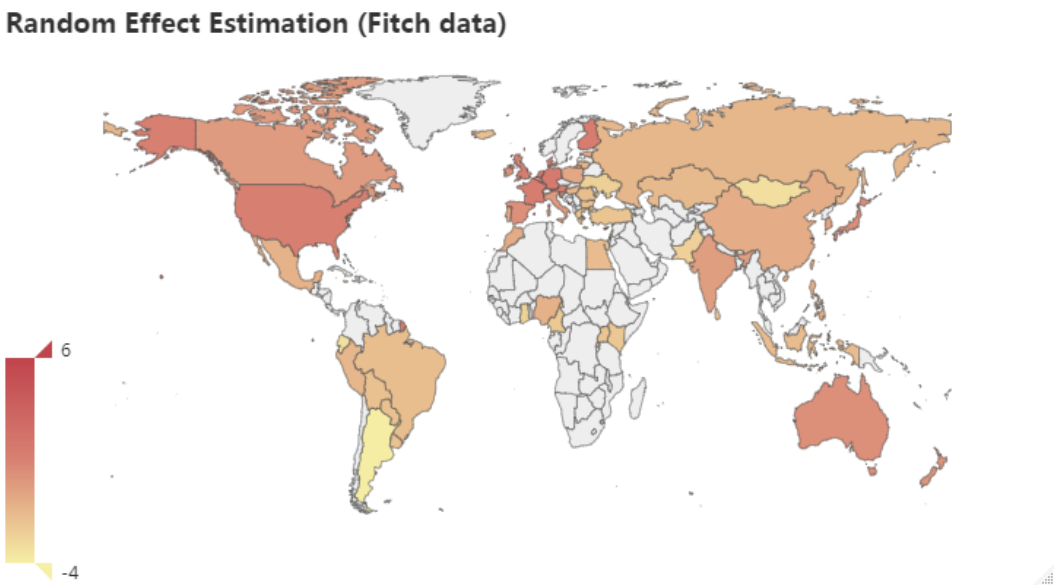


Figure 5.11: Estimation of Random Effect with Fitch data

Slightly different from the result of S&P result, United States receive a relatively-high rating assignment. And countries in Northern Europe, Oceania and Middle East receive high-level of subject-specific parameter.

With the help of histogram and clustered figure 5.12, we get information about the distribution of data.

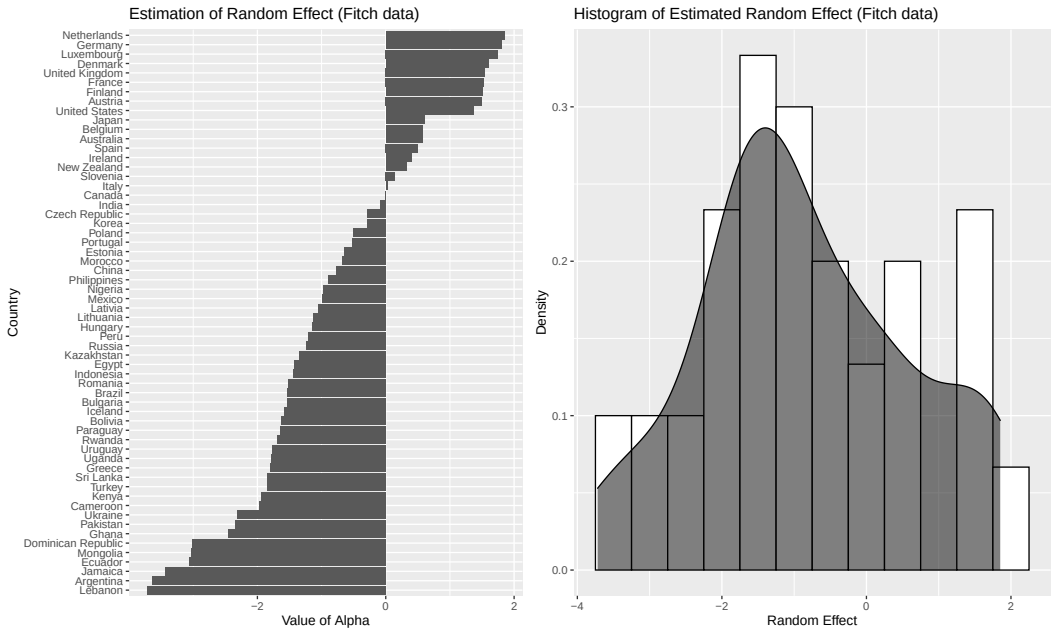


Figure 5.12: Histogram of Random Effect with Fitch data

When we focus on the results based on Moody and Fitch data, there is something worthy noted. From the histogram figures 5.10 and 5.12, we can see there is an extra peak area in density curve. And they correspond to a certain set of countries, as Austria, Denmark, Finland, France and United Kingdom. This show the agencies give highly similar weight to these countries. We can see these countries share similar economic and financial performance, political structure and geopolitical location. Moreover, there is also bias problems shown in the result as fixed effect ones done. For instance, Moody gives a relative-high random effect value to China, comparing with S&P and Fitch's rating assignment.

5.4 Feedback Effect

In feedback effect analysis, we focus on how the rating history movement influence the rating assignment. We introduce the feedback effect as indicator variable in the empirical analysis. As the initial step in analysis, we use linear regression to investigate the relationship between rating history movement

and rating assignment. In table 5.10, the result show that economic and financial variables have the similar influence on rating level as linear regression results shown in the first section. Of the coefficients, GDP per capita, real GDP, GDP growth rate, fiscal balance, and the indicator variables for economic development all have the positive signs and are statistically significant, on the contrary, inflation, debt ratio and the indicator variables for default history show negatively statistically significance. Under different agencies data analysis, the coefficient on the external balance are statistically insignificant, coming with the unexpected sign. Consider some reality case as reference, the economy or market force poor sovereign rating into seemingly strong external balance position, for example, during Argentine great depression in 2001-2003, the nation's external balance show surplus position, as well as the Russian financial crisis in 1998. Therefore, although the rating agencies might assign numerical weight to some variables in determining rating assignment, there is no systematic relationship between these variables and rating assignment.

Table 5.10: Estimation of linear regression, with S&P, Moody and Fitch Data.

Begin of Table			
	S&P's Rating	Moody's Rating	Fitch's Rating
Intercept	-9.186 *** [3]	-11.811 ***	-12.868 ***
	(-25.806) [4]	(-22.726)	(-37.276)
Feedback Effect			
One-period Upgrade	-0.072	-0.013	-0.037
	(-0.651)	(-0.101)	(-0.289)
One-period Downgrade	-0.872 ***	-0.841 ***	-1.026 ***
	(-8.138)	(-6.513)	(-7.806)
Two-period Upgrade	-0.104	-0.057	-0.130
	(-0.939)	(-0.454)	(-1.019)
Two-period Downgrade	-0.851 ***	-0.907 ***	-1.043 ***
	(-7.841)	(-6.900)	(-7.934)
Econ. & Fin. Covar.			
Per capita income	0.961 ***	1.012 ***	1.349 ***
	(34.341)	(32.470)	(42.624)
Real GDP	0.229 ***	0.305 ***	0.263 ***
	(21.218)	(25.359)	(19.659)
GDP growth rate	0.038 ***	0.041 ***	0.020 ***

[3] . Significant at the 10 percent level; * Significant at the 5 percent level
** Significant at the 1 percent level; *** Significant at the 0.1 percent level
[4] The t-statistics are in parentheses

Continuation of Table 5.10			
	S&P's Rating	Moody's Rating	Fitch's Rating
	(7.866)	(7.730)	(3.417)
Inflation	-0.028 ***	-0.003 **	-0.020 ***
	(-16.796)	(-4.439)	(-10.505)
Fiscal Balance	0.046 ***	0.045 ***	0.031 ***
	(10.039)	(8.798)	(5.517)
External Balance	0.009 **	0.005	-0.003
	(3.258)	(1.627)	(3.414)
Debt ratio	-0.004 *	-0.004 **	-0.005 **
	(-2.373)	(-2.893)	(-3.096)
Indicator Covar.			
Default History	-1.041 ***	-0.830 ***	-1.716 ***
	(-10.026)	(-7.506)	(-12.604)
Industrialization	0.881 ***	0.913 ***	0.320 ***
	(18.664)	(17.560)	(4.835)
Least Developed Country	-0.950 ***	-0.991 **	-1.626 ***
	(-6.033)	(-2.747)	(-6.831)
Adjusted R-squared	0.712	0.664	0.665
Standard error	0.972	1.068	1.050
End of Table			

The result shown in table 5.10 also concludes that the negative rating movement in history have a significant influence on rating assignment, i.e, the rating downgrade lead sovereign issuers receive lower rating level. These rating history movement includes the movement in previous quarter and two quarters ago.

Chapter 6 Conclusion

In linear regression analysis, the results for all rating agencies allow us to conclude that all the coefficients have the expected signs, which is discussed in 'determinants in sovereign ratings' section, except for the external balance. And, most of the coefficients are indeed statistically different from zero. To more specific, better economic and financial performance, which can be concluded from GDP, GDP growth or government budget balance, provide strong support for a higher rating level. On the contrast, a poor performance in monetary policy or external debt, or worse that a default event happened before, will lead sovereign issuers to get lower rating levels. Then we turn to the indicator variable. The development classification variables, as industrialization and non-LDC, have positive effects. And, the membership in some cooperation organization makes the sovereign to get a higher rating level. It's noteworthy that, the external balance is no longer statistically significant in some rating agencies' rating system, comparing with the existing study or analysis. What's more, the results assesses that whether S&P, Moody or Fitch are using the same explanatory variables when they indicative rating levels. This is tested by pooling the data for all ratings and by running a common linear regression.

In the result of fixed effect analysis, we can see that, first, the estimated parameter values of economic and financial variables have the same signs, with the comparison to linear regression model. We can conclude these variables affect the sovereign rating level assignment in a expected way, as we summarized in linear analysis part. Moreover, the estimated parameter for the indicator variables show difference among the rating agencies. It reveals that the sign and weight, which are defined by rating agencies, of these geopolitics and development variables are different. Then we work on the subject-specific parameter, in this model which is estimated as a constant term. As a general conclusion, the sovereign issuers which are assigned similar rating level share noticeable characters. A relatively high rating level will be assigned to a sovereign issuer, which have a outstanding performance in economics and finance, stable political structure and monetary policy or geopolitics patterns. On the contrary, most of countries in South American, Africa and Asia get

relatively low rating level. There are some noticeable points in the empirical results. First, the sovereign issuers, which have defaulted on their bond or experienced financial crisis, as Spain, Greece, Russia, Brazil and Argentina, receive low fixed effect on rating level. We can conclude that, the stability in economic and financial performance and bond solvency play significant roles in rating system. Second, the biggest economy, United States, China and Japan, receive relatively low fixed effect on rating level.

Comparing with the results from linear regression and fixed effect analysis, the random effect result shows that the estimation values of each common parameter have same signs. In particular, GDP growth and fiscal balance have positive relationship with the numerical rating level. On the contrary, a higher inflation, external balance, debt service ratio or a default history event relate to lower rating. Similar as the conclusion in fixed effect analysis, the result reveals the sovereign issuers which are assigned similar rating level share noticeable characters, as performance in economics and finance, stable political structure and monetary policy, geopolitics patterns or bond solvency. And, there are two more conclusions in random effect analysis. First, there is "bias" problem in rating assignment. Second, the random effect estimation shows regional differences.

In the feedback effect analysis, with results from linear regression analysis which takes rating history movement into consideration, we can conclude that the downgrades in rating history have negative effect on rating assignment. And these rating downgrade includes negative rating movement in last quarter of the year and the movement two quarters ago.

Bibliography

Afonso, A. (2003, March). Understanding the determinants of sovereign debt ratings: Evidence for the two leading agencies. *J Econ Finan 27*(1), 56–74.

Afonso, A., P. Gomes, and P. Rother (2011, January). Short- and long-run determinants of sovereign debt credit ratings. *Int. J. Fin. Econ. 16*(1), 1–15.

Archer, C. C., G. Biglaiser, and K. DeRouen (2007, April). Sovereign bonds and the "democratic advantage": Does regime type affect credit rating agency ratings in the developing world? *Int. Organ. 61*(2), 341–365.

Bissoondoyal-Bheenick, E., R. Brooks, and A. Y. N. Yip (2006, September). Determinants of sovereign ratings: A comparison of case-based reasoning and ordered probit approaches. *Global Finance Journal 17*(1), 136–154.

Burda, M. and C. Wyplosz (2013). *Macroeconomics: A European Text*. OUP Oxford.

Cantor, R. and F. Packer (1996, December). Determinants and impact of sovereign credit ratings. *Fixed Income 6*(3), 76–91.

Cosset, J.-C. and J. Roy (1991, March). The determinants of country risk ratings. *Journal of International Business Studies 22*(1), 135–142.

Demidenko, E. (2013, August). *Mixed Models: Theory and Applications with R*. John Wiley & Sons.

Eaton, J. (1996). Sovereign debt, reputation and credit terms. *International Journal of Finance & Economics*.

Fitch (2012). Sovereign rating criteria. *New York: Fitch Ratings*.

Fuchs, A. and K. Gehring (2017, March). The home bias in sovereign ratings. *J. Eur. Econ. Assoc.*.

Greene, W. H. (2011, November). *Econometric Analysis*. Pearson Higher Ed.

Haque, N. U., M. S. Kumar, N. Mark, and D. J. Mathieson (1996, December). The economic content of indicators of developing country creditworthiness. *IMF Econ Rev 43*(4), 688–724.

Heakal, R. (2014). Exploring the current account in the balance of payments.

Henderson, C. R. (1975, June). Best linear unbiased estimation and prediction under a selection model. *Biometrics 31*(2), 423–447.

Hill, P., R. Brooks, and R. Faff (2010, June). Variations in sovereign credit quality assessments across rating agencies. *Journal of Banking & Finance 34*(6), 1327–1343.

Horrigan, J. O. (1966). The determination of Long-Term credit standing with financial ratios. *Journal of Accounting Research 4*, 44–62.

Hsiao, C. (2003, February). *Analysis of Panel Data by Cheng Hsiao*. Cambridge University Press.

Hu, Y.-T., R. Kiesel, and W. Perraudin (2002). The estimation of transition matrices for sovereign credit ratings. *Journal of Banking & Finance 26*(7), 1383–1406.

International Monetary Fund. Statistics Dept. (2014, June). *External Debt Statistics: Guide for Compilers and Users*. International Monetary Fund.

Lai, T. L. and H. Xing (2008). *Statistical Models and Methods for Financial Markets*. Springer Texts in Statistics.

Lando, D. and T. M. Skødeberg (2002, March). Analyzing rating transitions and rating drift with continuous observations. *Journal of Banking & Finance 26*(2), 423–444.

Larraín, G., H. Reisen, and J. Von Maltzan (1997). Emerging market risk and sovereign credit ratings.

Lee, S. H. (1993, April). Are the credit ratings assigned by bankers based on the willingness of LDC borrowers to repay? *J. Dev. Econ. 40*(2), 349–359.

Moody, J. (1918). *Moody's Analyses of Investments*. Analyses Publishing Company.

Moody, J. (1920). *Moody's Analyses of Investments. Part III, Government and Municipal Securities*. New York: Moody's Investors Service.

Moody, J. (1922). *Moody's Analyses of Investments and Security Rating Books*. Moody's Investors Service.

Moody, S. (2004). Moody's rating symbols & definitions. *Moody's Investors Service, Report 79004* (08), 2004.

Moody's Corporation (2001). *Moody's Country Credit Statistical Handbook*. Moody's Investors Service.

Moon, C. G. and J. G. Stotsky (1993). Testing the differences between the determinants of moody's and standard & poor's ratings an application of smooth simulated maximum likelihood estimation. *J. Appl. Econometrics*.

Standard and Poor's Corporation (1979, November). *Standard & Poor's ratings guide: corporate bonds, commercial paper, municipal bonds, international securities*. McGraw-Hill.

Standard&Poor (2017). Sovereign rating methodology. *Criteria — Governments — Sovereigns: Sovereign Rating Methodology*.

Thompson, W. E., J. V. Hickey, and M. L. Thompson (2016, April). *Society in Focus: An Introduction to Sociology*. Rowman & Littlefield.

Truglia, V. J. (1999). Moody's sovereign ratings: A ratings guide. *Moody's Rating Methodology Handbook: Sovereign,–()*.

Ul Haque, N., N. C. Mark, and D. J. Mathieson (1998, April). The relative importance of political and economic variables in creditworthiness ratings.